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Solitary Wave Solutions to the Sharma-Tasso-Olver Equation and the Similar Hirota-Satsuma KdV System through the Modified Simple Equation Method

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Authors' contributions

This work was carried out in collaboration between both authors. Author HJD obtained the traveling wave solution using the MSE method, and wrote the manuscript. Author LZL reviewed and edited the manuscript. Both authors read and approved the final manuscript.

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Abstract

In this paper, the modified simple equation (MSE) method is applied to find the exact solutions for the Sharma-Tasso-Olver (STO) equation and the similar Hirota-Satsuma KdV system. The MSE method is an effective method in investigating exact solitary wave solutions to nonlinear evolution equations (NLEEs) in the field of applied mathematics, mathematical physics and engineering. And it is very direct and effective.

Keywords: Sharma-Tasso-Olver equation; the modified simple equation method; solitary wave solutions.

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1 Introduction

Nonlinear partial differential equations (NLPDEs) are often used to describe the physical system, and the NLEEs have become a very useful tool for describing natural phenomena of science and engineering models. In the recent years, construction of exact solutions, in particular, solitary wave solutions of NLPDEs is an important significance of nonlinear science. Accordingly, a number of methods for finding exact solutions of NLPDEs have been proposed, such as the Jacobi elliptic function method [1], the extended tanh-function method [2], the inverse scattering transform [3], the F-expansion method [4], the (G'/G)-expansion method [5], the Backlund transformation method [6], the sine-cosine method [7], the first integration method [8], the auxiliary equation method [9], the homogeneous balance method [10], and others.Of course, there are a lot of important papers published in recent years which using the modified simple equation method such as [11]-[21]. The objective of this article is to apply the MSE method to construct the exact solutions for NLEEs in mathematical physics such as the STO equation and the similar Hirota-Satsuma KdV system. The rest of this article is organized as follows: In section 2, we give brief descriptions of the MSE method. In section 3 and section 4, we employ the MSE method to STO equation and the similar Hirota-Satsuma KdV system, and we describe the pictures of those Solitary Wave Solutions by MATLAB. In section 5, we summarize and discuss our results.

2 The Method

A.J.M. Jawad, M.D. Petkovic and A. Biswas described the MSE method in [22], the main steps are summarized in the following steps:

For a given NLPDE in the form

$$H(u, u_x, u_t, u_{xx}, u_{tt}, \cdots) = 0. \tag{2.1}$$

In general, the left hand side of Eq.(2.1)is a polynomial in u and its various derivatives.

Step 1: We seek the traveling wave solution of(2.1)in the form:

$$u(x,t) = y(z); z = k(x - \omega t), \tag{2.2}$$

where k and ω are constants to be determined later. Under the transformation(2.2), Eq.(2.1)is reduced to an ordinary differential equation(ODE)

$$F(y, y', y'', \cdots) = 0. \tag{2.3}$$

Where F is a polynomial in $u(z)$ and its derivatives, wherein $u'(z) = \frac{du}{dz}$.

Step 2: We suppose that Eq.(2.3)has the solution in the form:

$$y(z) = \sum_{i=1}^{N} a_i [\frac{s'(z)}{s(z)}]^i, \tag{2.4}$$

where $a_i(i = 0, 1, 2, \cdots, N)$ are constants to be determined, such that $a_N \neq 0$, and $s(z)$ is an unidentified function to be evaluated. In Jacobi elliptic function method, sine-cosine method, tanh-function method, Exp-function method etc, the solutions are proposed in terms of some functions established in advanced. But in the MSE method, $s(z)$ is neither pre-defined nor a solution of any prescribed differential equation. Therefore, it is not possible to conjecture from earlier what kind of solutions one may get through this method. This is the individuality and distinction of this method. Therefore, some new solutions might be found by this method.

Step 3: The positive integer N appearing in Eq.(2.4) can be determined by taking into account the homogeneous balance between the highest order nonlinear terms and the derivatives of the highest order occurring in Eq.(2.3).

Step 4: Substitute (2.4) into (2.3), calculate all the necessary derivatives $y, y', y'', ...$ and then account the function $s(z)$. As a result of this substitution, we get a polynomial of $s'(z)/s(z)$ and its derivatives. In this polynomial, we equate with zero all the coefficients of it. This operation yields a system of equations which can be solved to find $a_i (i = 0, 1, 2, \cdots, N)$, and $s(z)$.Consequently, we can get the exact solution of $Eq.(2.1)$.

3 Application of the Method to the STO Equation

Let us consider the STO equation

$$u_t + \alpha(u^3)_x + \frac{3}{2}\alpha(u^2)_{xx} + \alpha u_{xxx} = 0. \tag{3.1}$$

In order to find the exact solitary wave solutions of the equation, we use the wave variable

$$u(x, t) = y(z), z = k(x - \omega t), \tag{3.2}$$

where k and ω are constants to be determined. The wave transformation (3.2) reduces (3.1) into the ODE in the following form:

$$(-k\omega)y' + 3\alpha k y^2 y' + 3\alpha k^2[(y')^2 + yy''] + \alpha k^3 y''' = 0, \tag{3.3}$$

where prime denotes derivatives with respect to z. Now, integrating Eq.(3.3) with respect to z, we get a new ODE in the form:

$$(-\omega)y + \alpha y^3 + 3\alpha k y y' + \alpha k^2 y'' = 0. \tag{3.4}$$

In order to determine the value of N, balancing y^3 and y'' gives $N = 1$. Therefore, the solution of Eq.(3.4)takes the form:

$$u(x, t) = y(z) = a_0 + a_1\left(\frac{s'}{s}\right), \tag{3.5}$$

where a_0 , a_1 and a_2 are constants to be determined later such that $a_1 \neq 0$, and $s(z)$ is an unknown function. The derivatives of y are given in the following :

$$y' = \frac{a_1 s''}{s} - \frac{a_1(s')^2}{s^2}, \tag{3.6}$$

$$y'' = \frac{2a_1(s')^3}{s^3} - \frac{3a_1 s' s''}{s^2} + \frac{a_1 s'''}{s}, \tag{3.7}$$

$$y^3 = \frac{a_1^3(s')^3}{s^3} + \frac{3a_0 a_1^2(s')^2}{s^2} + \frac{3a_1 a_0^2 s'}{s} + a_0^3. \tag{3.8}$$

Substituting the values of y , y' , y'' and y^3 into Eq. (3.4) and then setting each coefficients of $s^{-j}, j = 0, 1, 2, \cdots$ to zero, we obtain a system of algebraic equations for a_0, a_1 and $s(z)$:

$$(-\omega)a_0 + \alpha a_0^3 = 0, \tag{3.9}$$

$$(-\omega)a_1 s' + 3\alpha a_0^2 a_1 s' + 3\alpha k a_0 a_1 s'' + \alpha k^2 a_1 s''' = 0, \tag{3.10}$$

$$3\alpha a_1^2 a_0 (s')^2 + 3\alpha k\left(a_1^2 s' s'' - a_0 a_1 (s')^2\right) - 3\alpha k^2 a_1 s' s'' = 0, \tag{3.11}$$

$$\alpha a_1^3 (s')^3 - 3\alpha k a_1^2 (s')^3 + 2\alpha k^2 a_1 (s')^3 = 0. \tag{3.12}$$

From Eq.(3.9), we obtain

$$a_0 = 0, \pm\sqrt{\frac{\omega}{\alpha}}.$$

In order to simplify the process, we assume that α , ω is greater than zero. And Eq.(3.12), yields

$$a_1 = k, 2k,$$

since $a_1 \neq 0$.

Therefore, for the values of a_0, a_1, there arise the following cases:

Case 1: When $a_0 = 0$, from Eqs. (3.10) and (3.11), we obtain

$$s(z) = c_1 e^{\sqrt{\frac{\omega}{\alpha k^2}}z} + c_2 e^{-\sqrt{\frac{\omega}{\alpha k^2}}z},$$

where c_1 and c_2 are integration constants.

Substituting the values of a_0, a_1 and $s(z)$ into Eq.(3.5) we obtain the following exponential form solution:

$$y(z) = k\frac{\sqrt{\frac{\omega}{\alpha k^2}}c_1 e^{\sqrt{\frac{\omega}{\alpha k^2}}z} - \sqrt{\frac{\omega}{\alpha k^2}}c_2 e^{-\sqrt{\frac{\omega}{\alpha k^2}}z}}{c_1 e^{\sqrt{\frac{\omega}{\alpha k^2}}z} + c_2 e^{-\sqrt{\frac{\omega}{\alpha k^2}}z}}, \tag{3.13}$$

or

$$y(z) = 2k\frac{\sqrt{\frac{\omega}{\alpha k^2}}c_1 e^{\sqrt{\frac{\omega}{\alpha k^2}}z} - \sqrt{\frac{\omega}{\alpha k^2}}c_2 e^{-\sqrt{\frac{\omega}{\alpha k^2}}z}}{c_1 e^{\sqrt{\frac{\omega}{\alpha k^2}}z} + c_2 e^{-\sqrt{\frac{\omega}{\alpha k^2}}z}}. \tag{3.14}$$

Simplifying the required solution (3.13) and (3.14) , we derive the following close-form solution of the STO equation (3.1):

$$y(z) = \sqrt{\frac{\omega}{\alpha}}\frac{(c_1 - c_2)\cosh\sqrt{\frac{\omega}{\alpha k^2}}z + (c_1 + c_2)\sinh\sqrt{\frac{\omega}{\alpha k^2}}z}{(c_1 + c_2)\cosh\sqrt{\frac{\omega}{\alpha k^2}}z + (c_1 - c_2)\sinh\sqrt{\frac{\omega}{\alpha k^2}}z}, \tag{3.15}$$

or

$$y(z) = 2\sqrt{\frac{\omega}{\alpha}}\frac{(c_1 - c_2)\cosh\sqrt{\frac{\omega}{\alpha k^2}}z + (c_1 + c_2)\sinh\sqrt{\frac{\omega}{\alpha k^2}}z}{(c_1 + c_2)\cosh\sqrt{\frac{\omega}{\alpha k^2}}z + (c_1 - c_2)\sinh\sqrt{\frac{\omega}{\alpha k^2}}z}. \tag{3.16}$$

Solution (3.15) , (3.16) is the generalized solitary wave solution of the STO equation. Since c_1 and c_2 are arbitrary constants, one might arbitrarily choose their values. Therefore, choose $c_1 = c_2$, we obtain the following solutions:

$$u_1(x,t) = 2\sqrt{\frac{\omega}{\alpha}}\tanh\sqrt{\frac{\omega}{\alpha}}(x - \omega t), \tag{3.17}$$

$$u_2(x,t) = 4\sqrt{\frac{\omega}{\alpha}}\tanh\sqrt{\frac{\omega}{\alpha}}(x - \omega t). \tag{3.18}$$

Choose $c_1 = -c_2$, we obtain the following solutions:

$$u_3(x,t) = 2\sqrt{\frac{\omega}{\alpha}}\coth\sqrt{\frac{\omega}{\alpha}}(x - \omega t), \tag{3.19}$$

$$u_4(x,t) = 4\sqrt{\frac{\omega}{\alpha}}\coth\sqrt{\frac{\omega}{\alpha}}(x - \omega t). \tag{3.20}$$

The other choices of c_1 and c_2, we might obtain much new and more general exact solutions, for succinctness, we do not give all of them.

Case 2: When $a_0 \neq 0$, we discuss this case $a_1 = 2k$. We assume that α , ω is greater than zero, and when $k = 1$ or $k = \frac{1}{2}$, we can refer to this paper [23]. Solving Eq.(3.10) and (3.11), we get

$$s = c_2 + \frac{m}{l} e^{-lz}, \tag{3.21}$$

$$s' = -me^{-lz}, \tag{3.22}$$

where $l = \frac{-\omega}{\alpha a_0 k}$, $m = \frac{k}{a_0 c_1}$. and c_1 and c_2 are constants of integration. Substituting the Eq.(3.21), (3.22) into the Eq.(3.5), we get the following solution

$$y = a_0 + (2k) \frac{(-k\sqrt{\frac{\alpha}{\omega}})c_1 e^{\frac{\omega}{\alpha a_0 k} z}}{c_2 + (\frac{-\alpha k^2 c_1}{\omega}) e^{\frac{\omega}{\alpha a_0 k} z}}. \tag{3.23}$$

Then simplify it, we obtain

$$y = a_0 + \frac{(-2k^2)\sqrt{\frac{\alpha}{\omega}} c_1 \Big(\cosh \frac{\omega}{2\alpha a_0 k} z + \sinh \frac{\omega}{2\alpha a_0 k} z \Big)}{(c_2 - \frac{\alpha k^2 c_1}{\omega}) \cosh \frac{\omega}{2\alpha a_0 k} z - (c_2 + \frac{\alpha k^2 c_1}{\omega}) \sinh \frac{\omega}{2\alpha a_0 k} z}. \tag{3.24}$$

Solution (3.24) is the generalized solitary wave solution of the STO equation. Since c_1 and c_2 are arbitrary constants, one might arbitrarily choose their values. Therefore, when $a_0 = \sqrt{\frac{\omega}{\alpha}}$, choose $c_2 = \frac{\alpha k^2 c_1}{\omega}$, we obtain the following solution:

$$u_5(x,t) = 2\sqrt{\frac{\omega}{\alpha}} + \frac{\omega}{\alpha} \coth \frac{1}{2} \sqrt{\frac{\omega}{\alpha}} (x - \omega t). \tag{3.25}$$

Choose $c_2 = -\frac{\alpha k^2 c_1}{\omega}$, we obtain the following solution:

$$u_6(x,t) = 2\sqrt{\frac{\omega}{\alpha}} + \frac{\omega}{\alpha} \tanh \frac{1}{2} \sqrt{\frac{\omega}{\alpha}} (x - \omega t). \tag{3.26}$$

When $a_0 = -\sqrt{\frac{\omega}{\alpha}}$, choose $c_2 = \frac{\alpha k^2 c_1}{\omega}$, we obtain the following solution:

$$u_7(x,t) = \frac{\omega}{\alpha} \coth \frac{-1}{2} \sqrt{\frac{\omega}{\alpha}} (x - \omega t). \tag{3.27}$$

Choose $c_2 = -\frac{\alpha k^2 c_1}{\omega}$, we obtain the following solution:

$$u_8(x,t) = \frac{\omega}{\alpha} \tanh \frac{-1}{2} \sqrt{\frac{\omega}{\alpha}} (x - \omega t). \tag{3.28}$$

Now we give some figures of the solutions with different parameters of Case 1.

4 Application to the Similar Hirota-Satsuma KdV System

Let us consider the similar Hirota-Satsuma KdV system

$$\begin{cases} u_t = \frac{1}{2} u_{xxx} + 3uv_x, \\ v_t = uu_x. \end{cases} \tag{4.1}$$

We referred to Eq.(4.1) as the similar Hirota-Satsuma KdV system, which was derived by Hirota and Satsuma. The system typically describes an interaction of two long waves with different dispersion relations. Now, we use the MSE method to find the solitary wave solutions to the system (4.1). Let

$$u(z) = u(x,t), v(z) = v(x,t), z = k(x - \omega t),$$

system (4.1) becomes

$$\begin{cases} (-k\omega)u' = \frac{1}{2} k^3 u''' + 3kuv', \\ (-k\omega)v' = kuu'. \end{cases} \tag{4.2}$$

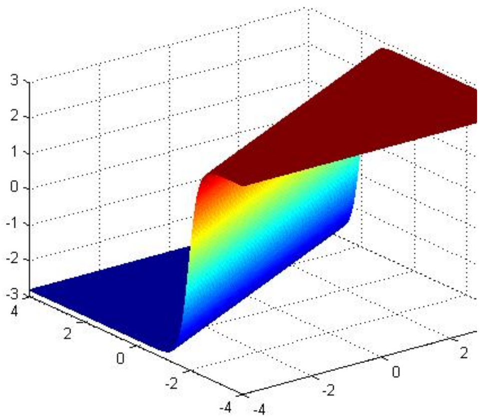

Fig. 1. Shape of (3.17)**,when** $\omega = 2, \alpha = 1$

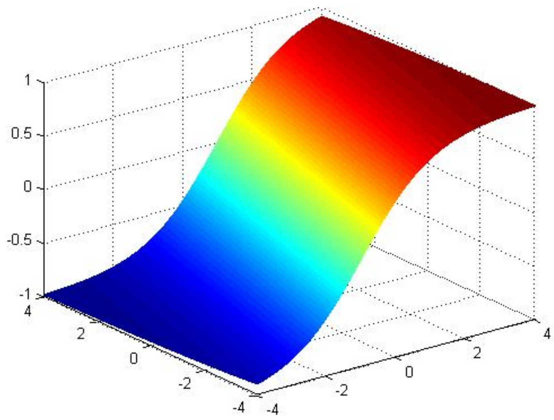

Fig. 2. Shape of (3.17)**,when** $\omega = 1/4, \alpha = 1$

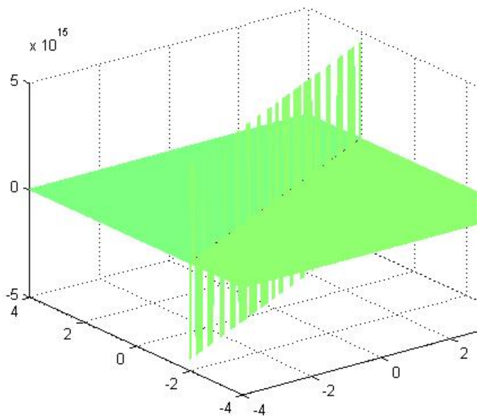

Fig. 3. Shape of(3.19)**,when** $\omega = 2, \alpha = 1$

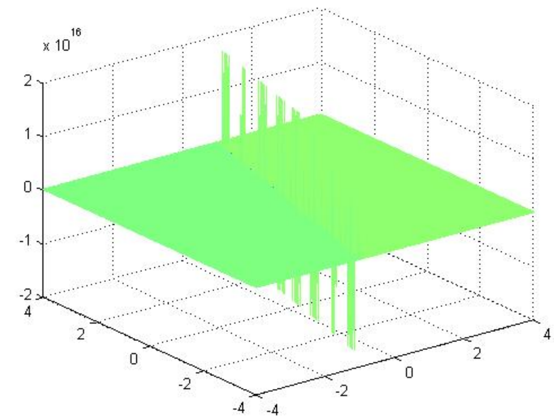

Fig. 4. Shape of (3.19)**,when** $\omega = 1/4, \alpha = 1$

Integrating the second equation of (4.2) with respect to z, and neglecting the constant of integration, we obtain

$$v = \frac{-u^2}{2\omega}. \tag{4.3}$$

Substituting Eq.(4.3) into system (4.2) we get

$$(k\omega)u' + \frac{1}{2}k^3u''' - \frac{6k}{\omega}u^2u' = 0. \tag{4.4}$$

Balancing the highest order derivative u''' and nonlinear term u^2u', we get $N = 1$. Thus, the solution of Eq.(4.4) becomes

$$u(z) = a_0 + a_1\left(\frac{s'}{s}\right). \tag{4.5}$$

where a_0, a_1 are constants to be determined later such that $a_1 \neq 0$, and $s(z)$ is an unknown function to be determined.It is easy to see that

$$u' = a_1\left(\frac{s''}{s}\right) - a_1\left(\frac{s'}{s}\right)^2, \tag{4.6}$$

$$u'' = a_1\left(\frac{s'''}{s}\right) - 3a_1\left(\frac{s's''}{s^2}\right) + 2a_1\left(\frac{s'}{s}\right)^3, \tag{4.7}$$

$$u^3 = a_1^3\left(\frac{s'}{s}\right)^3 + 3a_1^2a_0\left(\frac{s'}{s}\right)^2 + 3a_1a_0^2\left(\frac{s'}{s}\right) + a_0^3. \tag{4.8}$$

Integrating Eq.(4.4) with respect to z, we can easily get the following equation

$$(k\omega)u + \frac{1}{2}k^3u'' - \frac{2k}{\omega}u^3 = 0. \tag{4.9}$$

Now substituting the values of u , u'' , u^3 in Eq.(4.9) and then equating the coefficients of s^0, s^{-1}, s^{-2} and s^{-3} to zero, we respectively obtain

$$k\omega a_0 + \Big(\frac{-2k}{\omega}\Big)a_{0^3} = 0, \tag{4.10}$$

$$k\omega a_1s' + \frac{1}{2}a_1k^3s''' - \Big(\frac{-6k}{\omega}\Big)a_1a_0^2s' = 0, \tag{4.11}$$

$$\frac{3}{2}k^3a_1s's'' + \Big(\frac{6k}{\omega}\Big)a_0a_1^2(s')^2 = 0, \tag{4.12}$$

$$k^3a_1(s')^3 - \Big(\frac{2k}{\omega}\Big)a_1^3(s')^3 = 0. \tag{4.13}$$

Solving Eq.(4.10), we get

$$a_0 = 0, \pm\frac{\sqrt{2}}{2}\omega.$$

Solving Eq.(4.13), we have

$$a_1 = \pm\frac{\sqrt{2\omega}}{2}k.$$

Solving Eq.(4.11) and (4.12), we get

$$s = c_2 + \frac{m}{l}e^{-lz}, \tag{4.14}$$

$$s' = -me^{-lz}, \tag{4.15}$$

where $l = \frac{6a_0^2-\omega^2}{2a_0a_1}$, $m = \frac{k^2\omega c_1}{4a_0a_1}$, and c_1 and c_2 are constants of integration.

Substituting the Eq.(4.14), (4.15) into the Eq.(4.5), we get the following solution

$$u = a_0 + a_1\Big(\frac{-me^{-lz}}{c_2 + \frac{m}{l}e^{-lz}}\Big). \tag{4.16}$$

When $a_0 = \pm\frac{\sqrt{2}}{2}\omega$, $a_1 == \pm\frac{\sqrt{2\omega}}{2}k$. Putting the values of a_0 , a_1 into Eq.(4.16) and simplify it, we obtain

$$u(z) = \frac{-\sqrt{2}}{2}\omega\left(1 - \frac{k^2c_1}{2\omega}\frac{\cosh(\frac{\sqrt{\omega}}{k}z) \pm \sinh(\frac{\sqrt{\omega}}{k}z)}{c_2\Big(\cosh(\frac{\sqrt{\omega}}{k}z) \mp \sinh(\frac{\sqrt{\omega}}{k}z)\Big) + \frac{c_1k^2}{4\omega}\Big(\cosh(\frac{\sqrt{\omega}}{k}z) \pm \sinh(\frac{\sqrt{\omega}}{k}z)\Big)}\right)$$

where $z = k(x - \omega t)$.

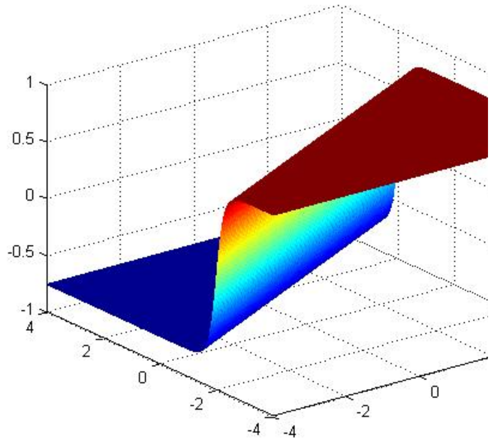

Fig. 5. Shape of (4.19)**,when** $\omega = 4$

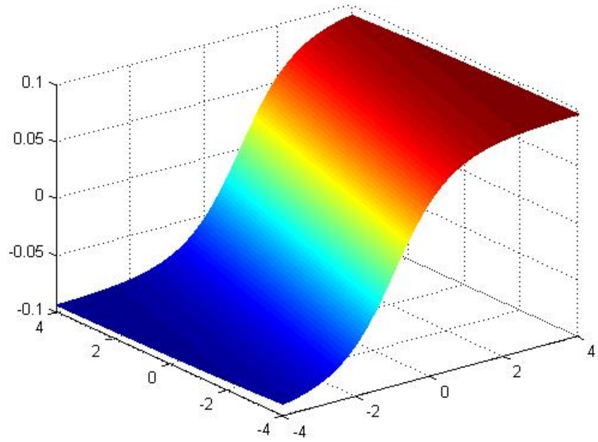

Fig. 6. Shape of(4.19)**,when** $\omega = 1/4$

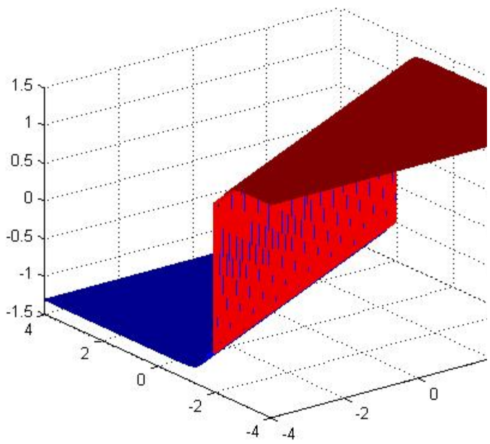

Fig. 7. Shape of (4.20)**,when** $\omega = 4$

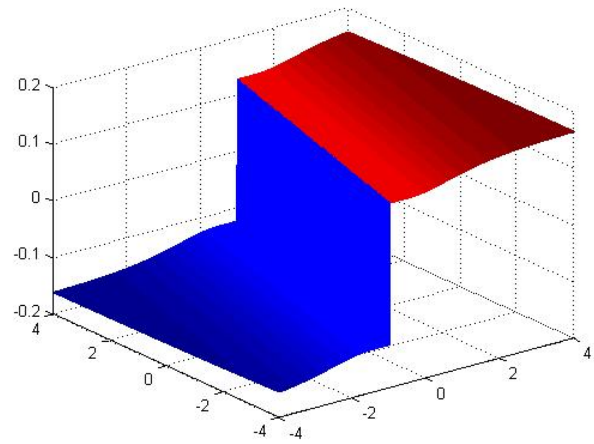

Fig. 8. Shape of (4.20)**,when** $\omega = 1/4$

We can freely choose the constants c_1 and c_2, for example, choose $c_2 = \frac{c_1 k^2}{4\omega}$, we get

$$u_1(x,t) = \pm\frac{\sqrt{2}}{2}\omega \tanh\sqrt{\omega}(x-\omega t). \tag{4.17}$$

Choose $c_2 = -\frac{c_1 k^2}{4\omega}$, we get

$$u_2(x,t) = \pm\frac{\sqrt{2}}{2}\omega \coth\sqrt{\omega}(x-\omega t). \tag{4.18}$$

Now, combined (4.17) and (4.18) with Eq.(4.3), we get

$$v_1(x,t) = \frac{1}{2}\omega \tanh^2\sqrt{\omega}(x-\omega t), \tag{4.19}$$

and

$$v_2(x,t) = \frac{1}{2}\omega \coth^2 \sqrt{\omega}(x - \omega t). \tag{4.20}$$

$(4.17) - (4.20)$ are the exact traveling wave solutions of the similar Hirota-Satsuma KdV system. Now we give several figures of the solutions with different parameters.

5 Conclusions

In this paper, we constructed some exact solutions of NLPDEs such as the STO equation and the similar Hirota-Satsuma KdV system. Since the considered equation have been shown to be applicable to many dynamics problems in physics, and the exact solutions will be helpful in related research and numerical studies. When the parameters receive special values, solitary wave solutions are derived from the exact solutions. We depict the graphs and have analyzed the solitary wave properties of the solutions for different values of physical parameters via the graphs.

Competing Interests

The authors declare that no competing interests exist.

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