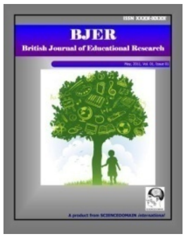

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The Interaction between Intuitive Interpretations, Linguistic Knowledge and Algorithmic Components in Children´s (Aged 7-9) Subtraction Errors

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ABSTRACT

Aims: In the present paper, we have examined the interaction between formal, intuitive and algorithmic knowledge in the solution for the subtraction operation (Fischbein, 1994).
Study design: Qualitative analysis.
Place and Duration of Study: Department of Didactic, Organization and Investigation Methods (University of Salamanca), between February 2005 and July 2006.
Methodology: We included the verbal protocols of nine children aged between seven and ten years old who solved a total of 180 subtractions were analyzed. The volume of data obtained via verbal protocols has allowed us to study the influence of the conceptual framework and the way in which the children interpret algorithmic process in the first stage of teaching.
Results: The results have allowed certain suggestions to be made with regard to the relation between formal, procedural and intuitive components that have a bearing on the generation of errors, and offer deep insights into how teachers influence the acquisition of mathematical concepts in the primary education stage.
Conclusion: we consider as influential in the origin of the subtraction error is that the child's intuitive interpretations formed in structural schemata, the vocabulary that forms part of these, and the semantic interpretation of zero uphold the sources that generate analogue transfer. It is important to consider its influence in order to improve the algorithmic teaching processes.

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Keywords: Algorithmic learning; semantic approximation to error; primary education; subtraction errors; empirical research; teaching practice; intuition; intuitive rules.

1. INTRODUCTION

Research in the field of systematic errors made during the learning of the subtraction algorithm comes from different theoretical perspectives. Firstly, the syntactic approach, related to VanLenh's Repair Theory, which has provided important results regarding the procedural mechanisms that govern the generation of systematic errors made in the process of solving a subtraction algorithm (Brown and Burton 1978; Brown and VanLehn 1982; VanLehn, 1982, 1983, 1986, 1987, 1990; VanLehn and Brown 1980; Young and O'Shea, 1981). This theoretical framework explains the learning process of cognitive skills in instructional settings (VanLehn, 1990, p. 12), and how and why we make mistakes. The theory suggests that when a procedure cannot be performed, an impasse occurs and the individual applies various strategies, which are called "repairs", to overcome the impasse. Some repairs generate incorrect results called "buggy" procedures. Repair theory assumes that people learn procedural tasks by induction and that bugs are introduced in the examples provided during the instruction or the feedback received while doing practice (Brown and VanLehn, 1980; VanLehn, 1990).

The second approach employed corresponds to the line of research which evaluates types of comprehension and the conceptual foundation that children acquire when learning the multiple digit subtraction algorithm.

A deep analysis of the latter approach has been carried out by authors such as Fuson, 1986, 1992; Fuson and Briars, 1990; Hiebert and Lefevre, 1986; Hiebert and Warne, 1996; Ohlsson and Rees 1991; Resnick, 1982, 1983; Resnick and Omanson, 1987; Steffe and Cobb, 1988. These authors consider the comprehension of the principles and concepts that organise the learning process as essential in order to avoid the errors which arise during learning. They therefore study the existence of a relation between understanding and skills in mathematics for children and instruction, and how these influence this relation (Baroody and Ginsburg, 1986; Cockbourn and Littler, 2008; Hiebert and Wearne, 1996; Kamii, 1985).

On this theoretical level we could also refer to other research focused on the influence of the semantic structures of arithmetic problems on the completion of those problems (Fischbein, et al., 1985). From the results of these studies we can deduce that the relationship between the formal intuitive components of mathematics learning (Fischbein, 1994) and linguistic knowledge is a key element in the generation of errors within problems containing the operations of multiplication and division. In a similar way, as regards the semantic structures, we have mentioned an important theoretical model that focuses its attention on the influence of semantic interpretations within arithmetic operations. Fischbein's Theory (1987, 1994, 1999) described the notion of intuitive models and the role they play in algorithmic knowledge.

"According to Fischbein, intuitive knowledge is a type of immediate, implicit, self-evident cognition that leads in a coercive manner to generalizations" (Tirosh and Tsamir, 2004, p.537). Fischbein considered the interaction between three of the components of mathematics knowledge: intuitive, formal and algorithmic. "The formal aspect refers to axioms, definitions, theorems and proofs. The algorithmic aspect refers to solving techniques and standard strategies. The intuitive aspect refers to the degree of subjective, direct acceptance by an individual of a notion, a theorem, or a solution…" (Fischbein, 1994, p.244), and it is organized in tacit models that Fischbein (1987) describes as imperfect mediators that lead to incorrect interpretations of the algorithm. Analogies play an important role in the construction of models. They are a source of models and may occasionally be a source of misconceptions (Fischbein, 1987).

At times, during algorithmic learning, a conflict takes place between the intuitive aspect and the formal interpretation of the procedure. Thus, on an educational level, algorithmic error could be predicted via this irrelevant interpretation of the procedure. "In this case, the teacher has to identify the intuitive tendencies of the student and to try to explain their sources" (Fischbein, 1999, p. 24). Likewise, Stavy and Tirosh (2000) stress the predictive power of these irrelevant interpretations of the process.

In this spirit, Tirosh, Tsamir, and Hershkovitz (2008) asserted that many of the errors made by students in subtraction can be explained by the influence of a number of intuitive rules. Such an assertion is part of the Intuitive Rules Theory (Stavy and Tirosh, 1996, 2000; Tsamir, 2005; Tirosh and Stavy, 1996, 1999), proposed to explain and predict inappropriate responses to a wide range of mathematics and science tasks. In this theory, the authors confirm that many students react by giving similar answers in a wide range of tasks that are not conceptually related, although they so have one component or external trait in common. From the analysis of these answers, they conclude that many of them can be explained by the influence of three rules: 'More A- more B', 'Same A- Same B'; 'Everything can be divided endlessly'. They consider these rules intuitive because students feel as if their explanations were self-evident and sufficient (Stavy et al., 2006). They likewise possess attributes of globality (the students tend to repeat them in different situations) and coerciveness, because the alternatives are often excluded as unacceptable (Stavy et al., 2006).

Tirosh and Stavy (1999) report that students' responses to a variety of mathematical comparison tasks were influenced by the intuitive rule 'Same A- Same B' in tasks where 'the two objects or systems to be compared were equal in respect to one quality or quantity (A1 = A2) but different in respect to another one (B1 = B2). In some of the tasks, the equality in quantity. A was perceptually or directly given. In other cases, the equality in quantity A could be logically derived. A common incorrect response to all these tasks, regardless of the content domain, was B1= B2 because A1 = A2' (Tirosh and Stavy, 1999, p. 62). They also affirm that this rule could be activated by a perceptual or logical input. On this point, they suggest that it could be innate and stress that it is reasonable to assume that children generalize such experiences into a universal maxim: 'Same A- Same B', because this rule is often applied in different situations within the school context. This fact would promote its generalization. In light of this theory, we suggest that algorithmic reasoning is affected by intuitive rules. Specifically, we believe that the rule 'Same A- Same B' plays an important role in this affirmation. In this study we propose an explicit reference to this rule when we analyse the incorrect generalizations made by the children regarding the concept of zero.

Other authors have informed of the intuitive component in algorithmic knowledge (Baroody, 1988; Gelman and Gallistel, 1978; Huttenlocher, et al., 1994; Resnick, 1987; Sander, 2001).
Sander (2001) agrees with Fischbein and believes that the interpretative aspects are essential in the generation of errors in vertical multi-digit subtraction. Sander's results confirm that errors are generated by a negative analogue transfer mechanism that occurs in educational environments whose sources are inadequate. He considers two sources that support the "Remove and Distance" analogy, integrated in the conceptual schema that

support the skill and used by children, either spontaneously or as the result of certain learning situations. According to Sander, interpretations based on these sources initially give rise to errors of a semantic nature. Subsequently, these errors influence procedural errors.

Previous ideas have led us to focus more on the process of mathematical knowledge transfer and its possible role in the generation of errors in subtractions. Some authors such as Brown and Clement, 1989; Clement, 1993; Duit, 1991; Fischbein, 1987; Gentner et al., 2003; Thagard, 1992; Sander, 2001; VanLehn, 1986, 1990; Zook, 1991; Zook and DiVesta, 1991 have studied this kind of transfer. Hiebert and Lefevre (1986) believe that this process facilitates the relation between conceptual and procedural knowledge. Fuson (1992), points out that it is very common for teachers to rely on school textbooks when introducing algorithms in mathematics classes. This may interfere with the children's ability to make generalisations. Likewise, VanLehn (1990) believes it is not only the examples given in the text books which interfere with the generalisation of the subtraction algorithm process, but also those offered by teachers or fellow students.

Consequently, the sources that feed the process of arithmetic knowledge transfer can be induced from the learning context. According to Fischbein (1999) and Tirosh and Stavy (1999), intuitions are sensitive to the influence of the context. If that is so, we should further investigate the nature of the mathematic processes, methods, and language used in algorithmic learning situations.

In this study, we have specifically focused on analyzing the influence of the semantic perspective on the origin of errors within the subtraction algorithm teaching and learning context, keeping in mind the theoretical contributions of Fischbein in particular.

In our research area, the results of leading studies define certain fundamental theoretical aspects, which constitute the central nucleus in the generation of errors made in subtractions. These are linked to the following affirmations: (I) the relationship between intuition and algorithmic knowledge is essential to study the nature of the errors (Fischbein, 1987, 1993, 1994; Tirosh et al., 2008). (II) Algorithmic subtraction learning occurs through inductive mechanisms, using examples (VanLehn and Brown, 1980), (VanLehn, 1986, 1990), via a learning process of analogy (Sander, 2001; VanLehn, 1986, 1990). (III) Errors arise from an investigative or heuristic process (VanLehn, 1990) during the resolution of a new problem and (IV) The nature of the error is related to the conceptual acquisition occurring in the first phase of the learning process and therefore, the mistake is due to a lack of understanding of the meaning of 0 and of the positional place-value concepts (Cockburn and Parslow-Williams, 2008; Fiori and Zuccheri, 2005; Fuson 1986, 1992; Fuson and Briars, 1990; Kamii, 1985; Resnick and Omanson 1987; López and Sánchez, 2007, 2009).

This last assertion is rooted in the contributions of the previously mentioned authors, who suggest that not all the theory should rest on the procedural component. Therefore, subtraction algorithm learning, as well as requiring a certain aptitude for logical reasoning, as highlighted by Piaget and Szeminska (1941), and a certain level of development, also involves comprehension of the procedure, and these authors propose a set of basic principles closely related to the acquisition of base-10 structures, on which the aforementioned comprehension is based. These are; *(i) Additive composition of quantities, (ii) Conventions of decimal place value notation, (iii) Calculation through partitioning and (iv) Recomposition and conservation of the minuend quantity* (Resnick and Omanson, 1987, p. 49).

These authors believe that the cause of errors does not reside in syntactic or procedural aspects, which are considered superficial. They would, in short, be the result of violating one or more of the previously mentioned principles which are a basic element in the conceptual structure of the procedure.

The conceptual structures are very well described by Resnick (1983), who differentiates between *relational knowledge*, made up of proto-quantitative structures, and *representational knowledge,* which includes knowledge of verbal counting. The relational structures are: *(i) Comparison,* through which children have at their disposal a series of terms such as more, less, smaller than, *(ii) Increase – reduction*, which allows them to determine changes in quantity and (*iii) Part – Whole*, which allows the whole to be divided into smaller parts.

The progressive integration of both types of knowledge facilitates the execution of the algorithms.

From our point of view, the intuitive component defined by Fischbein is related to the interpretations that children make of the conceptual and relational structures defined by Resnick, and it is right at this point where we locate the interaction between the intuitive, the formal and the procedural components in the processes of understanding the subtraction algorithm.

VanLehn (1982, 1990), considers an error to be an invention resulting from an investigation process carried out during the resolution of the subtraction, although we would be dealing with a heuristic strategy, influenced by conceptual knowledge and the child's primary intuitive interpretations of the algorithm.

The intuitive interpretations of algorithmic practice or performance when starting the subtraction learning process are especially relevant when they are applied to different situations (Ohlsson and Rees, 1991) through negative analogue transfer mechanisms (Fischbein, 1987; Sander, 2001). In Fischbein's words: "analogies may be the source of misconceptions when correspondences are assumed which in fact are not parts of the structural mapping between the two systems. Often such misconceptions will arise through an incompatibility between a formal property of the system being modelled and an intuitive property of the modelling representation, which is consciously or tacitly guiding the cognitive processes" (Fischbein, 1987, p. 142).

According to Gentner et al., (1997), the analogue transfer process would consist of a structural alignment between two mental representations with the aim of finding the maximum consistency between them. From our point of view, this process would facilitate the generation of errors during algorithmic learning, since they can serve to guide intuitive interpretation. This is because intuitive conceptual interpretations during the first stage of instruction play an important role in attempts to adapt previously known algorithmic procedures to solving new problems or novel situations.

We believe that instruction intervenes in the generation of errors via processes acquired by memorization, which are highly influenced by teaching that does not encourage understanding of conceptual knowledge (Baroody, 1988; Fuson, 1986, 1992; Kamii, 1985; Resnick, 1982, 1983; Resnick and Omanson, 1987). Thus, interaction processes determine the choice of resources that constitute the understanding of the skill (Bromme and Steinbring, 1994).

That is to say, like Fischbein (1987) and Sander (2001), we maintain that the sources of analogue transfer used by children are not appropriate. In other words, the conduct displayed by children is characterised by a lack of understanding of the formal component in multi-digit subtraction, and the intuitive interpretation of this formal-procedural component (Fischbein, 1999).

It generates errors of a semantic nature, which are based on an analogue transfer process, in the first stage of instruction. The child carries out one by one all the elements that form the set of arithmetic skills based on the superficial features of the examples used during instruction. These are transferred to new subtractions whose structures are different from the ones which were initially solved and which were useful as example prototypes during instruction.

However, by looking at the conceptual background in greater depth, in the present paper we report on the results obtained with regard to the influence of children's intuitive interpretations on the mechanisms underlying the operation and the procedure as such. With this aim, we take as referents, on the one hand, the examples used in the instruction (VanLehn, 1986, 1990), memorized as routines in the first learning phase, and on the other hand, a series of structural schemata or action schemas (Fischbein, 1999), that can be found at the root of the execution of arithmetic skills and are useful for establishing relations between the quantities that make up the operation, allowing the child to develop as they solve the problem.

These structural schemata are; *'Taking one part of the whole' and 'Covering a distance´'* (Resnick, 1983), and "intuitions depend on a structural schema…Intuitions may, sometimes, be related to adequate schemata, but, sometimes, they may be manipulated by non-adequate schemata…." (Fischbein, 1999, p. 44). The child's interpretations formed in these non-adequate structural schemata, and the vocabulary that form part of these, constitute the resources that generate analogue transfer. This being so, the reason why instruction promotes these resources is that sometimes it does not develop the understanding of the algorithmic principles and ignores the intuitive interpretations and beliefs of the students.

Finally, the aim of the present paper is to broaden the theoretical framework and demonstrate how, when children start to learn, they develop intuitive interpretations of the procedure based on a series of concepts or specific vocabulary organised within the conceptual field of subtraction.

In summary, a basic assumption, which will be described in the present paper, is that the child's intuitive interpretations formed in structural schemata, the vocabulary that forms part of these, and the semantic interpretation of zero constitute the base for sources that generate analogue transfer and they affect the origin of the subtraction error .

2. MATERIALS AND METHODS

2.1 Introduction

In order to confirm the aforementioned premises, we designed a 'pilot study' as part of a wider research framework. The main aim of this pilot study was to analyse the semantic nature of errors in subtractions with borrowings.

Before describing this pilot study, we set out some aims and significant results, which are useful as referents for the study, and that form part of the wider research subject.

The general aims of the research that this pilot study is linked to are the following; (i) To confirm the level of acquisition of the set of basic principles defined by Resnick and Omanson (1987), (ii) To study the generation of systematic errors in our educational context and understand the typology of these errors.

Some of the most significant results of this research are those obtained in relation to knowledge of Base-10. A sample of 357 primary school students between the ages of 7 and 12 were assessed with a multiple choice test made up of 10 items through which we aimed to authenticate if the children had acquired the basic concepts necessary to comprehend the subtraction process.

We obtained a very low percentage of right answers: 44.5% of the sample (N=357). Only 4.3% of the test population worked with knowledge related to place-value of the digits within the numbers. Control of the natural series of the digit, on the other hand, only yielded 21.5% of the correct answers (López and Sánchez, 2007, 2009).

Regarding the generation of systematic errors *1, we found higher percentages than in previous studies (VanLehn, 1990). Thus, 55.55% of students in the third grade (aged 8-9) showed systematic errors, 52.05% in the fourth grade (aged 9-10) and 26.66 % in the fifth grade (10-11).

Logically, these results led us to investigate the semantic aspects that configure conceptual knowledge and its development in the classroom in the first phase of algorithmic acquisition. This phase corresponds to the second and third grade of primary education in our country (children aged 7-9). With that in mind we developed the pilot study described below.

2.2 Objectives of the Pilot Study

i. To analyse the semantic nature of errors in subtraction with borrowing.
ii. To study the intuitive tendencies related to the subtraction process
iii. To identify the structural schemata on which the intuitive tendencies are based and to analyse how the language used influences these structures.

2.3 Context of the Study

The context in which we carried out our research is located in a western province of Spain. The population used in the sample is made up of four schools. For the pilot study described in this paper, we took as a reference one of the schools, which we have named “centre 2”. This school is located in the mountainous area of the province of Salamanca, which is undergoing growing depopulation and has a low birth rate. The school fulfils a particular characteristic due to the small number of children in the school, only 18 in total. These children are taught by the same teacher from the beginning of their schooling, which is when conceptual structures for multi-digit numbers are configured.

From the interviews carried out with the teacher, we concluded that the methodology used in the classroom was primarily textbook based. The subtraction algorithm learning process is carried out through a sequence of lessons which cover the 2nd and 3rd grades of primary education, that is to say, children aged from 7 to 9. A new aspect related to the acquisition of the procedure is introduced into each lesson: two-column subtraction, three-column subtraction and borrowing…, and practice exercises taken from the book or proposed by the teacher are done as well. Generally, the topics are taken from textbooks. That is to say, the teacher follows a traditional approach in which the algorithm is introduced in a directed way. Moreover, the research findings in the cross-sectional study of the errors made in this school revealed that these errors were mainly semantic in origin. Nonetheless, we wished to study their origins in greater depth as well as their possible relation to the context of instruction these children had received.

2.4 Subjects

A sample of 18 primary school students between the ages of 7 and 12 were tested with the VanLehn 20 subtractions test (VanLehn 1990, p. 170). This test is comprised of 20 multicolumn subtractions, seventeen of which are subtractions with borrowings. According to the author, this test has been carefully designed in order to obtain different errors (VanLehn1990, p. 193).

For a pilot study we called for all students from 2nd, 3rd, and 4th, (N=9, aged 7-10) grades of primary school, which made up school sample (2). These children had been instructed in algorithms from the outset by the same teacher, due to the geographic-administrative characteristics previously mentioned. All the children from the 2nd, 3rd and 4th grade (N=9) were tested with the test consisting of the 20 VanLehn subtractions, (VanLehn 1990, p. 170) and gave a spoken explanation of the way in which they resolved each subtraction. The nine children were chosen for three reasons: (i) they were in the academic grades where the first phase of subtraction with borrowings is taught, which is where the greatest number of errors of a semantic nature can be found, and (ii) they had been taught by the same teacher throughout their schooling.

This last reason was to provide us with some type of relation between linguistic knowledge, encouraged via instruction, and the development of the arithmetic skill to be established.

We obtained a considerable amount of verbal reports which were recorded and transcribed on a registration form with the instruments stated below.

2.5 Instruments

Taking as a reference the contribution of Olhson and Langley (1988, p. 47) in relation to the Newell and Simon (1972) diagnostic method, we used a transcription of verbal recording protocol whereby we transcribed the results of the verbal reports of the children from each of the 20 subtractions in the VanLehn (1990) test, which allowed us to describe the cognitive abilities and specific vocabulary entailed in solving the algorithm. The dimensions of the transcription protocol were:

- Part A: Identification Data: Code, age, centre, Start and finish time.
- Part B: description of cognitive conduct in the resolution of subtractions: Verbalization, Correct/error and type, Sequence of actions, Observations.

An example of Part B of the protocol is included below, where we can appreciate the “Smaller – from-larger” error transcription process in one of the students.

Table I. Example of "Smaller – from-larger" error transcription in the protocol

Subtraction	Verbal Reports	Correct/type of error	Sequence of actions	Observations
1564 -887= 1323	Subtraction no. 10: Four minus seven, three. Six minus eight, two. Five minus eight, three. One minus nothing, one.	Error: "Smaller from larger"	Four minus seven Writes result Next column Six minus eight Writes result Next column Five minus eight Writes result Next column One minus nothing Writes result	Structural schema: Less than. Nothing concept = blank space

2.6 Procedure

We employed a qualitative analysis for the study of the data obtained via the verbal reports. This methodology allowed us to study the natural language used in the process of solving the subtractions. We used technological means to make audio recordings, which were later transcribed in text format using the Sound Scriber computer programme.

In order to reduce the large volume of data, we drew up an ordered collection of information, presented in an operative and extensive way which would allow questions arising from the research to be resolved. We transformed the data obtained from the verbalisations with the coding function of the Matrix in the computer program Nud*ist 4.0. (example in Appendix 1), a reference for qualitative evaluation, and this allowed us to determine the semantic conceptualisation of zero and whether or not transfer sources, defined by Sander (2001) as "Distance and Remove", existed in the texts and which of these were most used by the children in the test.

In this process of reducing data, the strategy used was to establish categories (Metacategories: "Distance and Remove" and "Semantic Conceptualisation of Zero"). In order to analyse the number of text fragments included in each metacategory, we used enumeration units found, each unit being equivalent to a line of text. We assigned verbal codes to the different fragments of text which gave information on the metacategory contents with the aim of verifying the categorisation process carried out in the text. These codes were made up of the expressions used by the children when solving the procedure that forms part of the structural schemata on which the arithmetic skill is based.

Below is an example of the categories which have been analysed in each of the metacategories:

Table II. Example of categories and codes

	Metacategories		
	Metacategory: Distance	**Metacategory: Remove**	**Metacategory: Conceptualisation of 0**
Codes	Started with/finished with Counted up to I went from…to From…to	Less than Take from I take I had I took I subtracted from	There is nothing in the zero Zero has nothing Zero minus nothing Zero is worth nothing Zero = nothing

3. RESULTS

3.1 Analysis of the First Period of Subtraction Algorithm Acquisition

Before applying the previously defined instruments, using VanLehn's 20 subtraction test (1990) we checked what the frequency and typology of the most common errors were in the first phase of learning using a sample of 357 primary school students between the ages of 7 and 12. To do this we took as referents the 2nd and 3rd grade of Primary Education forming part of this sample (aged 7-9) (N=135). The results are presented in Table III, which shows the dominant errors in frequencies and percentages according to the terminology used by VanLehn (1990). We can see that the error categories Smaller- from-larger and Diff, 0-N=N have a very high number of occurrences; although the analysis of higher years at school shows that these disappear.

Table III. Distribution (in %) of occurrences of 8 errors with the greatest frequency in the 2nd and 3rd grade (aged 7-9). N= 135.

Error type	Example*[2]	2nd Grade(N = 63)		3rd Grade (N = 72)	
		Number of occurrences	%	Number of occurrences	%
Smaller-from-larger	81-38=57	119	18.25	20	5.18
Fact errors	7-3=5	44	6.75	35	9.07
Borrow-no-decrement	64-44=28	81	12.42	46	11.92
Diff, 0-N=N	80-27=67	63	9.66	30	7.77
Forget-borrow-over-blank	347-9=348	41	6.29	11	2.85
1-1=1-after-borrow	812-518=314	38	5.83	18	4.66
Borrow-from-zero-is-ten	604-235=479	19	2.91	39	10.10
Diff, 0-N=0	40-21=20	16	2.45	6	1.56
Other errors		231	35.43	181	46.89
Total		652	100	386	100

Table IV shows the same analysis as that used for Table III, but this time the analysis was performed on the 2nd and 3rd grade (aged 7-9) students of the sample a (N=9) taken for the pilot study described here.

Table IV. Distribution (in %) of occurrences of 8 errors with the greatest frequency in the 2nd and 3rd grade (aged 7-9) in school (2), where the pilot study was carried out

Error type	Example*[2]	2nd Grade (N = 2)		3rd Grade (N = 4)	
		Number of occurrences	%	Number of occurrences	%
Smaller-from-larger	81-38=57	0	0	30	43.47
Fact errors	7-3=5	1	3.22	0	0
Borrow-no-decrement	64-44=28	4	12.90	0	0
Diff, 0-N=N	80-27=67	0	0	9	13.04
Forget-borrow-over-blank	347-9=348	0	0	0	0
1-1=1-after-borrow	812-518=314	0	0	0	0
Borrow-from-zero-is-ten	604-235=479	1	3.22	2	2.89
Diff, 0-N=0	40-21=20	5	16.12	6	8.69
Other errors		20	64.51	22	31.88
Total		31	100	69	100

The results shown in Tables III and IV highlight the difference in frequency of the appearance of errors (Smaller- from-larger, Diff, 0-N=N and Diff, 0-N=0) between the two school grades (Table III). Likewise, in Table IV we show the frequency of appearance of errors such as Smaller-from-larger, Diff, 0-N=N y Diff, 0-N=0 in School (2) where out study was carried out. At this school we found a tendency for these errors to increase in 3rd grade. The "Other errors" category in Table IV includes errors such as: Borrow-across-zero (904-7=807), 1-1=0-after-borrow (812-518=314). The causes behind these errors will be discussed in the next section.

We can affirm that the errors are semantic in nature. This being the case, we believe that these first errors are not a result of an "impasse" (VanLehn 1982, 1990), but a consequence of the first *intuitive interpretation* formed in a non-adequate structural schemata of the procedure (Fischbein, 1987, 1994, 1999; Sander, 2001) and based on an inadequate understanding of the conceptual domain or formal knowledge of the algorithm. The general characteristics of the errors revolve around conducts that systematically have a bearing on situations with a higher level of cognitive difficulty and complexity in the sphere of the conceptual understanding of the procedure. This fact will be projected when executing the skill, giving rise to violations of the rules that govern the process.

Generally specking, as an effect of age and instruction an adequate acquisition of all the structural schemata of arithmetic skills is progressively acquired, and does away with these false conceptions, causing some of these errors to disappear by the 4th year of primary education. This idea allows us to confirm the existence of conceptual or semantic errors.

3.2 Intuitive Interpretations in the First Phase of Learning the Algorithm

As previously stated in section 2.6, in order to analyse the verbal reports, we identified various expressions (Sander, 2001) that children use in the resolution of the algorithm related to the structural schemata : '*taking one part of the whole' and 'going from one initial situation to another final one'*. We added the semantic interpretation of 0 as a collection void of elements (Baroody, 1988). These structural schemata are very well described by Resnick (1983). In order to analyse the arithmetic knowledge of the selected children, we took into account relational schemata: (i) *Comparison,* by which children have at their disposal a series of terms such as more, less, smaller than, *(ii) Increase – reduction,* which allows them to determine changes in quantity and (iii) *Part – Whole,* which allows the whole to be divided into smaller parts.

The specific vocabulary of these schemata influences the child's initial interpretations of the subtraction procedure.

In order to obtain information on these interpretations, a total of 329 units of text were analysed, each unit corresponding to a line of text taken from the children's verbalisations during the execution of the 20 subtractions. We analysed, in total, the verbal reports of the nine children selected from school (2) for 180 subtractionsFor the data analysis we used Frequency Tables and commentary of the textual units in each of the three established meta-categories: *'Distance, Remove and Conceptualisation of the Zero'.* In the data shown in Table V, below, we have set out the expressions or categories which we identified most frequently in the text and which we can meta-categorize into two sources of analogue transfer, based on the transfer sources that Sander (2001) defined as *'Distance and Remove'*. In Table V, what Resnick (1983) defines as the relational knowledge of the subjects is fundamentally limited in our sample to the meta-category *'Moving'* within the semantic categories 'Less than' (53.49%), 'Take'(20.12%) and finally 'from…to' (19.14%). That is to say, if we add the percentages of the meta-category 'Remove' which is adapted to the structural schemata 'taking a part of the whole', we could conclude that it is the most common analogue transfer source in the sample.

If we advance a further step, however, and analyse what conceptual interpretation the children have of zero, we can see with greater clarity that the semantic or conceptual interpretation together with the interpretation of the procedure as "taking one part of the whole" has a significant influence on the production of errors.

3.3 Example of the Intuitive Rule 'Same A- Same B'

From the analysis of the verbalisations and the results we offered in the previous table, we have deduced that the concept *'nothing'* is intimately associated with zero in the sample. According to Haylock and Cockbourn (2003) this fixation on the idea that zero is nothing is a consequence of the emphasis on the cardinal aspect of number. Together with Kulm, (1985 cited by Baroody, 1988), we believe that such a coincidence in the verbalisations of all the children in the sample must be due to the fact that in the initial learning of zero as an intuitive estimation of a void set of elements the children use the example: '*zero is nothing*' or '*zero has nothing*', which the children memorized without completely understanding it, and applied it, through the analogue transfer mechanism, to all the zeros of the algorithm we evaluated. Thus, the intuitive estimation of zero depends on a structural schema that is adequate only for a set void of elements, but not adequate when zero is a positional holder. This may constitute an explicit example of the intuitive rule 'Same A- Same B' described by Tsamir (2005). We suggest that the incorrect generalisation of the initial intuitive estimation of zero as a set void of elements is analogous to the incorrect generalisation of zero as lacking place-value in base-10.

Table V. Distribution (in %) of occurrences of the meta/category Distance and Removing (N=9) Subtractions analysed = 180.

Meta-category: Distance				Meta-category: Remove			
Categories	**Number of occurrences**	**%**	**Total units of text**	**Categories**	**Number of occurrences**	**%**	**Total units of text analysed**
Started with/finished with	1	0.30	329	Less than	176	53.49	329
Counted up to	1	0.30	329	Take from	12	3.65	329
I went from…to	1	0.30	329	I take	66	20.12	329
From…to	63	19.14	329	I had I took	15	4.57	329
				I subtracted from	4	1.21	329

Table VI. Distribution (in %) of appearance of the meta-category: Conceptualisation of 0. Semantic interpretation of zero, (n=9). Subtractions analysed = 180.

Meta-category: Conceptualisation of 0			
Categories	**Number of occurrences**	**%**	**Total units of text analysed**
There is nothing in the zero	1	0.30	329
Zero has nothing	1	0.30	329
Zero minus nothing	8	2.40	329
Zero is worth nothing	2	0.61	329
Zero = nothing	199	60.48	329

This can be seen in the examples of some of the subtractions shown below:

Subtraction n^o. 17: (702-108=604, here I borrowed one and from eight to twelve I got four; here I didn't borrow anything; because it was zero and zero and zero, gave me zero; I borrowed one from seven; leaving me with six).
Subtraction n^o.14: (102-39=137, two minus nine seven, nothing minus three, one minus nothing one).

Subtraction n^o.14: (102-39=137, two minus nine seven, zero minus three, one minus nothing one).

3.4 Analysis of the Erroneous Concepts Developed Via the Transfer Process

In the above examples we can see that the student's answers possessed attributes that were self-evident, sufficient and global, since the students tended to repeat them in different situations, and coercive; because the alternatives are often excluded as unacceptable, as indicated by Stavy et al., (2006).

A mechanical use of analogy therefore occurs, through the student's inability to apply concepts from an analogous domain to a designated domain (Brown and Clement, 1989; Clement, 1993; Duit, 1991; Thagard, 1992; Zook, 1991; Zook and Digesta, 1991).

The erroneous concepts developed via the transfer process, embodied in the transfer source *"take one part of the whole",* of this first conceptual intuitive interpretation promotes the use of the concept *"nothing"* with the same semantic meaning in all situations. This situation can be difficult to rectify and in our case culminates in the generation of algorithmic errors. That is to say, during the instruction of these children, the subtraction algorithm had been associated with a non-significant learning of the place-value of the figures and the rule of transforming zero in subtractions with borrowings. These children consider zero to be a void set of elements and lacking place-value in base-10. This characteristic has a negative influence on the execution of the procedure by disarranging or disturbing the rules that govern the algorithmic process and generating errors, which as we will see below are of a semantic nature.

We are convinced that the children from our sample learnt the algorithm without understanding its structural principles, limiting themselves to mechanically following the rules set out by the teacher (Baroody and Ginsburg, 1986). They interpret multi-digit numbers as single-digit numbers. There is, therefore, a lack of understanding of place–value concepts (Fuson, 1992; Kamii, 1985).The teaching methodology used would contribute to certain errors being produced throughout the whole stage of Primary Education because the teacher has not identified the intuitive interpretations of the student, or tried to explain their sources (Fischbein, 1999).

These errors would be registered within the transfer source "*take from*", and would therefore be the result of the child's conceptual background from which the erroneous interpretation of the procedure was formed.

In Table VII shown below, we can see the conceptual interpretation of the most common transfer source in the development of this type of error.

Table VII. Distribution (in %) of the occurrences of the most common errors in school (2). School grades analysed 2nd, 3rd, 4th, 5th, and 6th. Children aged 7-12. (N=18). Subtractions analysed = 360

Error	%	Interpretation from the Transfer Source: Moving
Smaller-from-larger (e.g. 81-38=57)	26.60	When one takes or moves one part of the whole, the part is always smaller than the whole*3
1-1=0-after-borrow (e.g. 812-518=314)	15.23	If from one unit a unit is taken the result is 0. We believe the child takes the borrowing from the minuend and is therefore left with 0 and 0-N = 0. We are faced with the same situation as in error 0-N = 0. Because as these children conceive it, zero is nothing and therefore nothing can be taken from it. It would basically be the same conduct as in the following one.
Diff, 0-N=0 (e.g. 40-21 = 20)	12.38	It is impossible to take some part of the zero which for these children is made up of a set void of elements. Therefore, they opt for the interpretation of the error Smaller-from-larger. When one part of the whole is taken or moved, the part is always smaller than the whole.
Diff, 0-N=N (e.g. 80-27=67)	10.47	As zero is worth nothing, nothing can be taken or borrowed from it.
Borrow-across-zero (e.g. 904-237=577)	8.57	As zero is worth nothing, one cannot borrow across from it.
Other errors	26.75	

If we carefully analyse the conduct involved in the “Smaller-from-larger” error, it can be seen that the child has basically internalized the first part of the procedure as “take from”, but has not understood the directionality of the subtraction (Fuson, 1986, 1990, 1992). The child thinks that the “whole” is always the largest part; and this is an intuitive interpretation. Hence, they compare the digits unaware that the “whole” is in the subtrahend. So, they compare and subtract the minuend from the subtrahend and disregard the rule about the petition of “loans”.

According to Fischbein (1994), a conflict takes place between formal, intuitive and algorithmic components. This action means that all the principles that govern the process are violated, principles such as the place-value of digits, the conservation of the minuend, and so on. The result denotes a lack of conceptual knowledge, which has a bearing on the lack of corrective revision within the procedure (Ohlsson and Rees, 1991). This in turn is facilitated by this kind of knowledge. Because of this, the child is unable to act in an adequate manner in this discipline and resorts to the application of the heuristic for linear substitution (Gentner, 1983, 1997), or better stated, of an analogy based on intuitive interpretations of inadequate structural schemata.

The impoverished quality of conceptual knowledge present in this conduct is fundamentally associated with the acquisition of basic structures that constitute the numeric series and of rules that define the positional and grouping system.

Moreover, we believe that these errors are generated by the incorrect option of extrapolation, produced by an analogue transfer (Fischbein, 1987; Sander, 2001). Specifically in the bug "*smaller –from-larger*", the child carries out a linear extrapolation of a standard rule "take one part of the whole", which is transferred from a simple subtraction to a multicolumn subtraction with borrowing. That is to say, the child takes the rule learnt during instruction, which was generalized through examples, joint and isolated practice exercises and converted into a high level pattern, and transfers the rule to fit a new situation in which he/she has possibly been trained but not in a comprehensive manner. This characteristic significantly hinders the learning of the procedure or the acquisition of the whole pattern of rules governing the skill of subtraction with borrowing. As such, the student perceives the whole process as an *'isolated chain of conducts'* in the same way as a single column simple subtraction, unable to understand the hierarchically organised structures that prevail in the process of multicolumn subtraction with borrowing. The process of generation and production of this error prevails in this characteristic. We can also see that having deficient knowledge of the conceptual basis of zero, which is associated with a lack of "place-value" or *"positional holder"* (Resnick, 1987), leads to application of the incorrect reasoning that "one cannot subtract anything from nothing". We acknowledge the influence of language in the semantic understanding of zero and the intuitive interpretations in the first phase of learning. The children in the test sample use the word "nothing" with various semantic meanings.

4. CONCLUSION

According to Fischbein and Tirosh, Tsamir and Hershkovitz, students' intuitive ideas manipulate their formal reasoning and algorithmic procedures. In the study we present, the intuitive tendencies related to the subtraction process shown by students in the lower grades of Primary Education have been thoroughly examined. To explain these intuitions we have identified the structural schemata on which they are based and analysed the influence of the language used on such structures.

The research findings show that errors involve conducts that systematically have a bearing on the more complex cognitive phases of the process and are directly related to the comprehension of concepts, essential to the significant learning of arithmetic, basically in the field of principles governing the Base - 10 system (Fuson, 1992; Resnick, 1987). We have also demonstrated how intuitions children draw from the experiment interact with the formal knowledge of the algorithm. That is to say, the results obtained in the research confirm some of the contributions of Fischbein's theory and the Intuitive Rules Theory of Stavy and Tirosh (1996, 2000); Tsamir, (2005); Tirosh and Stavy (1996, 1999). One of the most relevant questions analysed is the relation between formal, procedural and intuitive knowledge in the subtraction algorithm.

We can also see the difficulty involved in the conceptual understanding of zero and the influence of the language used in the teaching process on this type comprehension. We believe that the vocabulary or specific linguistic understanding of the structural schemata which organize the skill is decisive in the generation of errors (see the results in Tables V and VI). As a result, we analysed the intuitive estimation of zero, which depends on a structural schema adequate only for a void set of elements, but inadequate when zero is a positional holder. This may constitute an explicit example of the intuitive rule 'Same A- Same B' described by Tsamir (2005). Thus, we have identified the structural schemata on which intuitions depend. This issue is of undoubted importance in the field of didactic procedures.

We have also verified that when children start to learn they construct intuitive interpretations of the practice or performance of algorithms. Going into more detail along this line, we have shown that these intuitive interpretations play an important role in the generation of errors and that they possess the attributes of being self-evident, sufficient, global and coercive.

We have shown, however, that the vocabulary inscribed in the structural schemata that support the execution of the skill is important in learning transfer or generalisation.

As such, we believe that by simplifying the acquisition model into two parts (the acquisition of the process and the syntactic interpretation of the same), VanLehn underestimated to a certain degree the formal and intuitive knowledge on which the procedural skill is based in the first stage of learning. It is this knowledge, as we understand it, which holds the whole conceptual framework that has a determining influence as a channelling element of the rules of the altered process. We are therefore faced with two types of errors: (I) those which at their genesis have as a matrix the conceptual or semantic influence closely related to inadequate and memory-based instruction and which do not take into account the possible interaction between formal and intuitive knowledge; (II) those which are the result of having generalized these erroneous procedures, which are internalized and become procedural ones, erroneously modifying the algorithm's syntax.

We consider that in the situations analysed, the conceptual background is not adequately registered in the mind of the learner. An analogue transfer process takes place, which we situate in the 4th grade of primary education, a level at which the conceptual teaching of the algorithm reaches its culmination and which we highlight as a dividing line between errors of a conceptual nature and those of a procedural nature.

In the present paper, we have shown that the interpretations and the resources used by the children when solving the algorithm should be taken into consideration by the teacher in the classroom because they have a decisive influence on the conceptual understanding of the algorithm. During instruction, teachers should pay particular attention to the language used in the teaching process, as well as to intuitive notions that children build during the learning process of the algorithm, which, as we have seen, are crucial in the acquisition of the structures that reinforce this competency.

To sum up, a basic assumption that we consider as influential in the origin of the subtraction error is that the child's intuitive interpretations formed in structural schemata, the vocabulary that forms part of these, and the semantic interpretation of zero uphold the sources that generate analogue transfer. It is important to consider its influence in order to improve the algorithmic teaching processes.

NOTES

1. We consider a mistake to be systematic when it is made in a high number of frequencies and throughout all the years.
2. Examples of errors taken from VanLehn, (1990 p.220-232).
3. Description of interpretation taken from (Sander 2001)

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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APPENDIX

Example of analysis with NUD *.IST 4.0: Analysis of code: "Zero minus nothing"

Search.txt page: 1 2/18/5 14:29:19

Q.S.R. NUD.IST Power version, revision 4.0

PROJECT: 1, 2:29 pm, Feb 18, 2005.

++
+++ Text search for 'cero menos nada'
+++ Searching document VOZ 1...

+++ Searching document VOZ 2...
32 RESTA 18: seis menos dos igual cuatro, cero menos cuatro igual cero; CERO MENOS NADA igual cero; dos
35 menos dos, cero; CERO MENOS NADA cero; uno menos nada uno.
+++ 2 text units out of 37, = 5.4%
+++ Searching document VOZ 3...
49 se puede hacer, así que diez menos dos, de dos a 10, van ocho, lo pongo y luego CERO MENOS NADA es cero, y
+++ 1 text unit out of 52, = 1.9%

+++ Searching document VOZ 4...
4 RESTA 4: cinco menos tres dos, CERO MENOS NADA cero, tres menos nada tres, ocho menos nada ocho.
20 RESTA 19: dos menos cuatro dos, uno menos uno cero, cero menos dos dos, CERO MENOS NADA cero, uno
22 RESTA 20: uno menos tres dos, cero menos cuatro cuatro, CERO MENOS NADA cero, ocho menos nada, ocho
+++ 3 text units out of 22, = 14%

+++ Searching document VOZ 5...
+++ Searching document VOZ 6...
+++ Searching document VOZ 7...

+++ Searching document VOZ 8...

7 RESTA 4: cinco menos tres me ha dado dos, y CERO MENOS NADA cero, tres menos nada tres y ocho menos nada
+++ 1 text unit out of 46, = 2.2%

+++ Searching document VOZ 9...
5 RESTA 4: cinco menos tres me ha dado dos, y CERO MENOS NADA cero, tres menos nada tres y ocho menos nada
+++ 1 text unit out of 31, = 3.2%

++
+++ Results of text search for 'cero menos nada':
++ Total number of text units found = 8
++ Finds in 5 documents out of 9 online documents, = 56%.
++ The online documents with finds have a total of 188 text units, so text units found in these documents = 4.3%.
++ The selected online documents have a total of 329 text units, so text units found in these documents = 2.4%.
++