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The Zubair-dagum Distribution

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Authors' contributions

This work was carried out in collaboration with the authors. Author ORU designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Author NPA managed the analyses of the study and the literature searches. Both authors read and approved the final manuscript.

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Abstract

This article examines the flexibility of the Zubair-G family of distribution using the Dagum distribution. The proposed distribution is called the Zubair-Dagum distribution. The various mathematical properties of this distribution such as the Quantile function, Moments, Moment generating function, Reliability analysis, Entropy and Order statistics were obtained. The parameter estimates of the proposed distribution were also derived and estimated using the maximum likelihood estimation method. The new distribution is right skewed and has various bathtub and monotonically decreasing shapes. Our numerical illustrations using two real-life datasets substantiate the applicability, flexibility and superiority of the proposed distribution over competing distributions.

Keywords: Dagum distribution; Zubair-G family; hazard rate function; entropy; order statistics.

1 Introduction

Distributions play important and significant roles in modelling several real-life problems. The Dagum distribution [1] is one of the distributions proposed to model income and wealth problems. Dagum [1] showed that this distribution is robust to heavy tails for wealth and income distributions. The Dagum distribution has been showed to be related to the Gini index [2]. Domma et al. [3] studied the properties and

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different methods of estimating the Dagum distribution. The Dagum distribution is widely known to have a monotonically decreasing and bathtub shaped hazard rate function [4]. This feature of the Dagum distribution has led to further extensions of the distribution to different studies. Shahzad and Asghar [5] proposed the Transmuted Dagum distribution. Other distributions includes the beta-dagum distributions [6], MC-Dagum distribution [7], Gamma-Dagum distribution [8], Weighted Dagum distribution [9], Eponentiated Kumaraswamy-Dagum distribution [10], Dagum-poisson distribution [11], Exponentiated generalized exponential Dagum distribution [12], Power log-Dagum distribution [13], exponentiated exponential-Dagum{Lomax} distribution [14].

These distributions have been applied to both censored and non-censored data and survival data from a reliability point of view across several fields such as economics, finance and actuarial sciences. The Dagum distribution [1] is defined thus;

Let X be a random variable that follows the Dagum distribution, then the c.d.f. of the three parameter Dagum distribution is obtained as

$$F_D(x) = \left(1 + \lambda x^{-\phi}\right)^{-\eta} \tag{1}$$

where $\lambda, \eta, \phi > 0$. λ is the scale parameter and η, ϕ are the shape parameters. The corresponding p.d.f. is obtained as

$$f_D(x) = \eta\lambda\phi x^{-\phi-1}(1 + \lambda x^{-\phi})^{-\eta-1} \tag{2}$$

The motivation of this work is to improve on the flexibility and characteristics of the Dagum distribution, and provide better fits to real-life datasets with respect to other extensions of the Dagum distribution.

One of the ways of improving a distribution is through generalization using generators. These generators allow baseline distributions to be extended while developing more realistic statistical models in different variety of applications Gomes-Silva et al. [15]. Recently, Ahmed [16] introduced a new family of distributions known as the Zubair-G distribution. This distribution is also a special sub-class of the Complementary exponentiated-G Poisson (CEGP) family of distribution earlier proposed by Tahir and Cordeiro [17].

The cumulative density function (p.d.f) and the probability density function (c.d.f) of the Zubair-G family of distribution is obtained as

$$F_Z(x) = \frac{\exp[\alpha G(x;\zeta)^2] - 1}{\exp[\alpha] - 1} \quad x > 0, \alpha > 0 \tag{3}$$

$$f_Z(x) = \frac{2\alpha f(x;\zeta) F(x;\zeta) \exp[\alpha F(x;\zeta)^2]}{\exp[\alpha] - 1} \quad x > 0, \alpha > 0 \tag{4}$$

where ζ is a vector of parameter/parameters for any baseline distribution.

The rest of the paper is organized thus, section 2 introduces the Zubair-Dagum distribution. Section 3 focuses on the mathematical properties of the proposed distribution. Section 4 focuses on the maximum likelihood estimation. Section 5 deals with the applications of the Zubair-Dagum distribution using some real-life datasets. Section 6 concludes the paper.

2 The Zubair-dagum Distribution

In this section, we derive and show graph of the cumulative density function (c.d.f) and the probability density function (p.d.f) of the Zubair-Dagum (ZD) distribution.

Let X be a random variable that follows the four parameter ZD distribution, then the c.d.f. of the four parameter ZD distribution is obtained by substituting (1) into (3) thus

$$F_{ZD}(x) = \frac{\exp\left[\alpha\left(1+\lambda x^{-\phi}\right)^{-2\eta}\right]-1}{\exp[\alpha]-1} \qquad x > 0, \alpha, \lambda, \varphi, \phi > 0 \tag{5}$$

where $x > 0, \alpha, \lambda, \eta, \phi > 0$. λ is the scale parameter and α, η, ϕ are the shape parameters. The corresponding p.d.f. is given by

$$f_{ZD}(x) = \frac{2\alpha\eta\lambda\phi x^{-\phi-1}(1+\lambda x^{-\phi})^{-2\eta-1}\exp\left[\alpha\left(1+\lambda x^{-\phi}\right)^{-2\eta}\right]}{\exp[\alpha]-1} \tag{6}$$

The figures below show the plots of the p.d.f and the c.d.f of the ZD distribution for different values of the parameters.

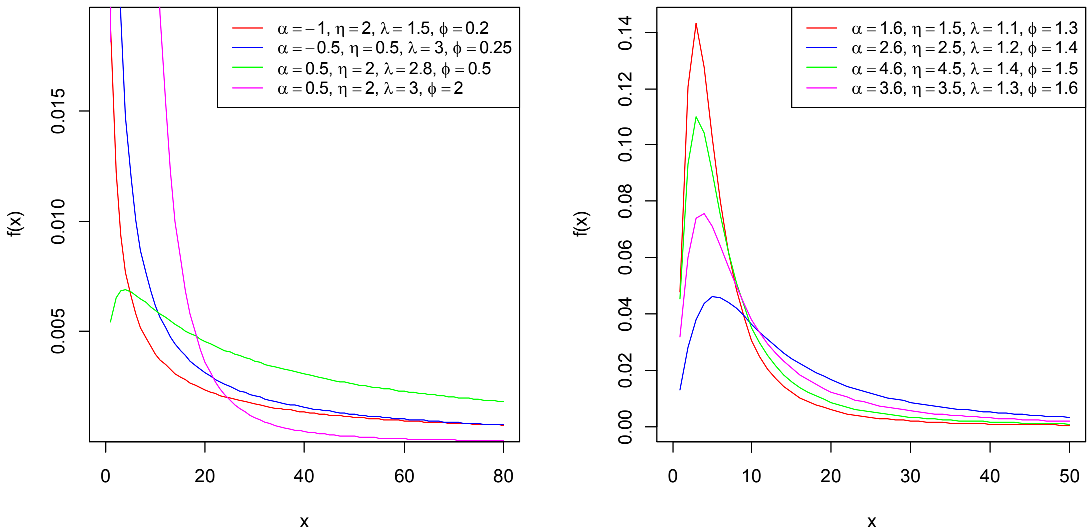

Fig. 1. The plots of the p.d.f of the Zubair-Dagum distribution

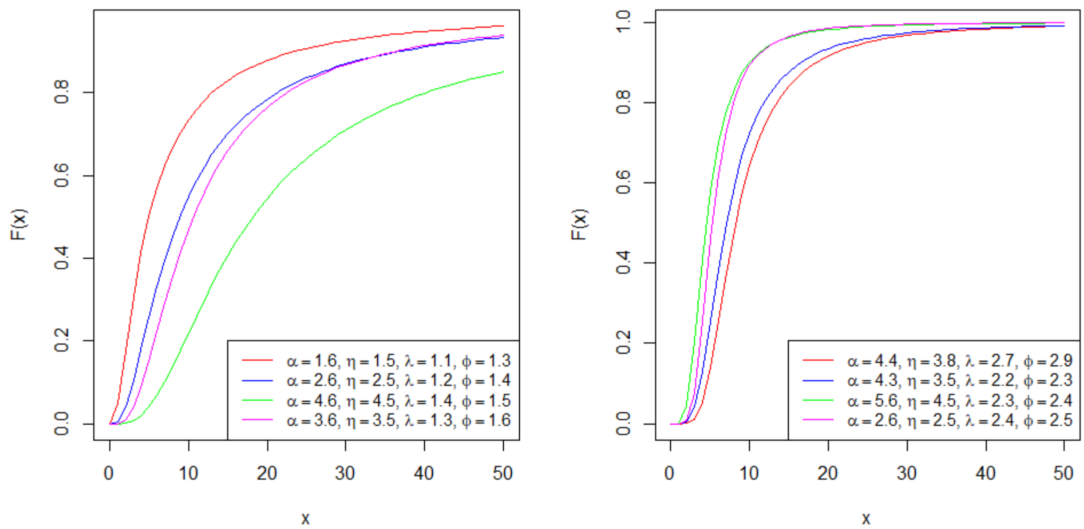

Fig. 2. The plots of the c.d.f of the Zubair-Dagum distribution

3 Mathematical Properties

In this section, some of the mathematical properties of the ZD distribution including the quantile function, moments and moment generating function, the order statistics, entropy and reliability analysis are discussed.

3.1 Quantile function of the Zubair-Dagum distribution

Given that a random variable X follows the ZD distribution, then the quantile function is given by

$$Q(w) = F^{-1}(w) \tag{7}$$

From the c.d.f. in (5), it follows that the quantile function of the ZD distribution is

$$x_w = \left\{\frac{1}{\lambda}\left[\left(\langle\frac{1}{\alpha}\ln(w[\exp(\alpha) - 1] + 1)\rangle^{1/2}\right)^{-1/\eta} - 1\right]\right\}^{-1/\phi} \tag{8}$$

3.2 Moments

Let X be a random variable that follows the ZD distribution with p.d.f (5), then the kth order moment about origin, $\mu_k^{'}$ is obtained by

$$\mu_r^{'} = \int_0^{\infty} x^k f_{ZD}(x)dx \tag{9}$$

$$\mu_r^{'} = 2\sum_{i=0}^{\infty}\frac{\alpha^{i+1}}{(\exp[\alpha]-1)i!}\int_0^{\infty} x^k f_{ZD}(x)F_{ZD}(x)^{2i+1}dx \tag{10}$$

where $f_{ZD}(x)$ and $F_{ZD}(x)$ are the p.d.f and c.d.f of the ZD distribution.

$$\mu_r^{'} = 2\sum_{i=0}^{\infty}\frac{\eta\lambda\phi\alpha^{i+1}}{(\exp[\alpha]-1)i!}\int_0^{\infty} x^{k-(1+\phi)}\left(1 + \lambda x^{-\phi}\right)^{-[2\eta(i+1)+1]}dx \tag{11}$$

$$\mu_r^{'} = 2\sum_{i=0}^{\infty}\frac{\eta\lambda^{\frac{k}{\phi}}\alpha^{i+1}}{(\exp[\alpha]-1)i!}B\left(1 - \frac{k}{\phi}, 2\eta(i+1) + \frac{k}{\phi}\right) \tag{12}$$

where $B(\cdot,\cdot)$ is the complete beta function.

3.3 Moment generating function

Let X be a random variable that follows the ZD distribution with p.d.f (5), then the moment generating function (m.g.f.) of X, $M_X(t)$ is obtained by

$$M_X(t) = E(e^{tx}) = \int_0^{\infty} e^{tx} f_{ZD}(x)dx \tag{13}$$

We write the m.g.f. of the ZD distribution in terms of the moments thus;

$$M_X(t) = 2\sum_{i=0}^{\infty}\frac{\eta\lambda^{\frac{k}{\phi}}\alpha^{i+1}}{(\exp[\alpha]-1)i!k!}B\left(1 - \frac{k}{\phi}, 2\eta(i+1) + \frac{k}{\phi}\right) \tag{14}$$

3.4 Order statistics

Given random samples, $X_1, X_2, X_3, \ldots, X_n$ from a ZD distribution. Let the corresponding order statistics be denoted by $X_{(1)}, X_{(2)}, X_{(3)}, \ldots, X_{(n)}$. Then, the probability density function, p.d.f of the pth order statistics, say $X = X_{(p)}$, is obtained by

$$f_p(x) = \frac{n! f_{ZD}(x)}{(p-1)!(n-p)!} \sum_{i=0}^{n-p} \binom{n-p}{i} (-1)^i F_{ZD}(x)^{p+i-1} \tag{15}$$

Substituting for the p.d.f and c.d.f, we have

$$f_p(x) = \frac{n! 2\alpha\eta\lambda\phi x^{-\phi-1}}{(p-1)!(n-p)!} \sum_{i=0}^{n-p} \binom{n-p}{i} (-1)^i \frac{(1+\lambda x^{-\phi})^{-2\eta-1} \exp\left[\alpha\left(1+\lambda x^{-\phi}\right)^{-2\eta}\right]}{\exp[\alpha]-1} \left[\frac{\exp\left[\alpha\left(1+\lambda x^{-\phi}\right)^{-2\eta}\right]-1}{\exp[\alpha]-1}\right]^{p+i-1} \tag{16}$$

$$f_p(x) = H_i x^{-\phi-1}(1+\lambda x^{-\phi})^{-2\eta-1} \exp\left[\alpha\left(1+\lambda x^{-\phi}\right)^{-2\eta}\right]\left[\exp\left[\alpha\left(1+\lambda x^{-\phi}\right)^{-2\eta}\right]-1\right]^{p+i-1} \tag{17}$$

where $H_i = 2\sum_{i=0}^{n-p} \frac{n!\alpha\eta\lambda\phi(-1)^i}{(p-1)!(n-p)![\exp[\alpha]-1]^{p+i-1}} \binom{n-p}{i}$

3.5 Entropy

The measurement of uncertainties associated with a random variable of a probability distributions is known as Entropy. Shannon Shannon, [18] and Rényi's entropy Rényi, [19] are widely used in the literature.

Given a random variable X, that follows a ZD distribution. The corresponding Rényi's entropy is given by

$$T_R(\gamma) = \frac{1}{1-\gamma} \log\left[\int_0^\infty f_{ZD}{}^\gamma(x) dx\right] \tag{18}$$

$$T_R(\gamma) = \frac{1}{1-\gamma} \log\left[\left[\frac{2\alpha\eta\lambda\phi}{\exp[\alpha]-1}\right]^\gamma \int_0^\infty x^{-\gamma(\phi+1)}(1+\lambda x^{-\phi})^{-\gamma(2\eta+1)} \exp\left[\alpha\gamma\left(1+\lambda x^{-\phi}\right)^{-2\eta}\right] dx\right] \tag{19}$$

Let $v = (1+\lambda x^{-\phi})^{-1}$, after some simplifications we obtain

$$T_R(\gamma) = \frac{1}{1-\gamma} \log\left[2^\gamma \sum_{i=0}^\infty \frac{t^i \alpha^{i+\gamma} (\eta\lambda\phi)^\gamma \lambda^{\frac{1-\gamma}{\phi}-\gamma}}{(\exp[\alpha]-1)^\gamma i! \phi} \int_0^\infty v^{\eta\gamma+\frac{1-\gamma}{\phi}-1} (1-v)^{\gamma-\frac{1-\gamma}{\phi}-1} dv\right] \tag{20}$$

$$T_R(\gamma) = \frac{1}{1-\gamma} \log\left[2^\gamma \sum_{i=0}^\infty \frac{t^i \alpha^{i+\gamma} (\eta\lambda\phi)^\gamma \lambda^{\frac{1-\gamma}{\phi}-\gamma}}{(\exp[\alpha]-1)^\gamma i! \phi} \frac{\Gamma\left(\eta\gamma+\frac{1-\gamma}{\phi}\right)\Gamma\left(\gamma-\frac{1-\gamma}{\phi}\right)}{\Gamma(\eta\gamma+\gamma)}\right] \tag{21}$$

Where $\gamma \longrightarrow 1$, the Rényi entropy converges to the Shannon entropy.

3.6 Hazard rate function

The conditional probability of failure of an item X, given it has survived up to the time t is known as the hazard rate function. It is obtained by

$$h(x) = \frac{f_{ZD}(x)}{1-F_{ZD}(x)} \tag{22}$$

$$h(x) = \frac{2\alpha\eta\lambda\phi x^{-\phi-1}(1+\lambda x^{-\phi})^{-2\eta-1} \exp\left[\alpha\left(1+\lambda x^{-\phi}\right)^{-2\eta}\right]}{\exp[\alpha]-\exp\left[\alpha\left(1+\lambda x^{-\phi}\right)^{-2\eta}\right]} \tag{23}$$

Fig. 3 below shows the plots of the hazard rate function for different values of the parameters.

3.7 Survival function

The probability that an item X does not fail prior to some time t is known as the Survival function. It is obtained by

$$s(x) = 1 - F_{ZD}(x) \tag{24}$$

$$s(x) = \frac{\exp[\alpha] - \exp\left[\alpha\left(1+\lambda x^{-\phi}\right)^{-2\eta}\right]}{\exp[\alpha]-1} \tag{25}$$

Fig. 4 below shows the plots of the survival function for different values of the parameters.

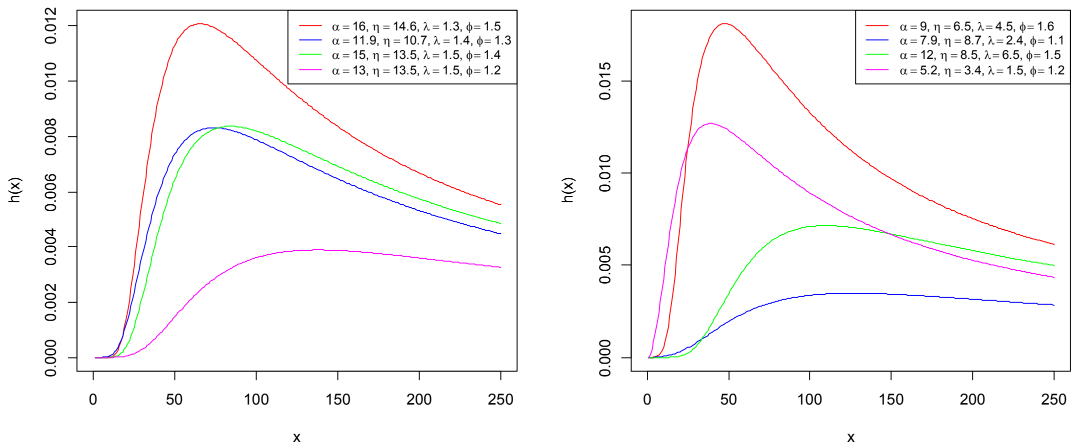

Fig. 3. The hazard rate function plots of the Zubair-Dagum distribution

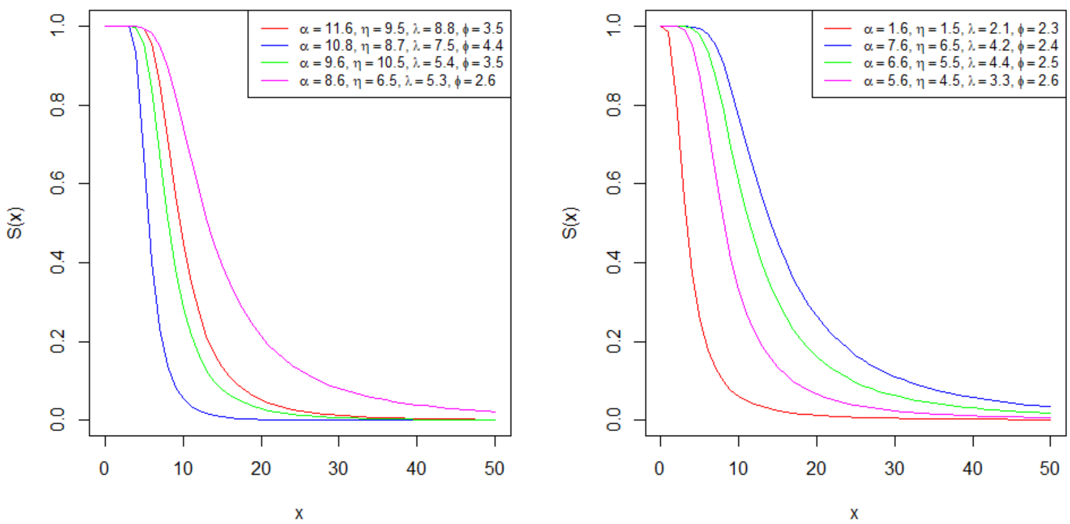

Fig. 4. The survival function plots of the Zubair-Dagum distribution

From Fig. 3 above, the hazard rate function exhibits shapes including monotonically decreasing, unimodal, and upside-down bathtub shapes. Also, from Fig. 4, the survival plots exhibit a monotonically decreasing shape.

4 Maximum Likelihood Estimation

Given a random sample of size n, $X_1, X_2, X_3, \ldots, X_n$ from the ZD distribution. The log-likelihood function of parameters can be obtained by

$$LL(x;\alpha\eta\lambda\phi) = \prod_{i=1}^{n} \ln F_{ZD}(x) \tag{26}$$

$$LL(x;\alpha\eta\lambda\phi) = \prod_{i=1}^{n} \ln\left\{\frac{2\alpha\eta\lambda\phi x_i^{-\phi-1}(1+\lambda x_i^{-\phi})^{-2\eta-1}\exp\left[\alpha\left(1+\lambda x_i^{-\phi}\right)^{-2\eta}\right]}{\exp[\alpha]-1}\right\} \tag{27}$$

$$LL(x;\alpha\eta\lambda\phi) = \ln\left\{\left(\frac{2\alpha\eta\lambda\phi}{\exp[\alpha]-1}\right)^n \prod_{i=1}^{n} x_i^{-\phi-1} \prod_{i=1}^{n}(1+\lambda x_i^{-\phi})^{-2\eta-1} \exp\left[\prod_{i=1}^{n} \alpha\left(1+\lambda x_i^{-\phi}\right)^{-2\eta}\right]\right\} \tag{28}$$

$$LL(x;\alpha\eta\lambda\phi) = n\ln(2\alpha\eta\lambda\phi) - (\phi-1)\sum_{i=1}^{n}\ln x_i - (2\eta+1)\sum_{i=1}^{n}\ln\left(1+\lambda x_i^{-\phi}\right) + \alpha\sum_{i=1}^{n}\left(1+\right.$$
$$\lambda xi - \phi - 2\eta - n \ln\exp \alpha - 1 \tag{29}$$

We take the partial differentiation of (29) w.r.t $\alpha, \eta, \lambda, and\ \phi$ and solve the nonlinear likelihood equations obtained so as to maximize the log-likelihood as shown below;

$$\frac{\partial LL}{\partial\alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n}\left(1+\lambda x_i^{-\phi}\right)^{-2\eta} - \frac{n\exp[\alpha]}{(\exp[\alpha]-1)} \tag{30}$$

$$\frac{\partial LL}{\partial\eta} = \frac{n}{\eta} - 2\sum_{i=1}^{n}\ln\left(1+\lambda x_i^{-\phi}\right) - 2\alpha\sum_{i=1}^{n}\left(1+\lambda x_i^{-\phi}\right)^{-2\eta}\ln\left(\alpha\sum_{i=1}^{n}\left(1+\lambda x_i^{-\phi}\right)\right) \tag{31}$$

$$\frac{\partial LL}{\partial\eta} = \frac{n}{\lambda} - 2\sum_{i=1}^{n}\frac{(2\eta+1)x_i^{-\phi}}{\left(1+\lambda x_i^{-\phi}\right)} - 2\alpha\eta\sum_{i=1}^{n} x_i^{-\phi}\left(1+\lambda x_i^{-\phi}\right)^{-2\eta-1} \tag{40}$$

$$\frac{\partial LL}{\partial\phi} = \frac{n}{\phi} - \sum_{i=1}^{n}\ln x_i + (2\eta+1)\sum_{i=1}^{n}\frac{\lambda x_i^{-\phi}\ln x_i}{\left(1+\lambda x_i^{-\phi}\right)} + 2\alpha\eta\lambda\sum_{i=1}^{n} x_i^{-\phi}\ln x_i\left(1+\lambda x_i^{-\phi}\right)^{-2\eta-1} \tag{41}$$

The nonlinear equations above are then equated to zero and solved simultaneously to obtain the estimates of the parameters. The solutions cannot be solved analytically hence we solve numerically using the Adequacy package of Marinho et al. [20] with “BFGS” algorithm. The R software [21] is used for this analysis.

5 Application

This section discusses the flexibility and superiority of the ZD distribution to some competing distributions using two real-life data sets. The first dataset consists of the failure times for 36 appliances subjected to an automatic life test. The dataset can be obtained from Lawless [22]. The second data consists of the time to failure of a 100 cm polyster/viscose yarn subjected to 2:3% strain level in textile experiment in order to assess the tensile fatigue characteristics of the yarn. The data set can be found in Quesenberry and Kent [23] and Pal and Tiensuwan [24].

These datasets are then fitted with the ZD distribution and compared with Dagum (D) distribution, Mc-Dagum (McD) distribution, Exponentiated Generalized Dagum (EGD) distribution and Exponentiated Kumaraswamy-Dagum (EKD) distribution with corresponding p.d.fs.

$$f_D(x) = \eta\lambda\phi x^{-\phi-1}(1+\lambda x^{-\phi})^{-\eta-1} \tag{42}$$

$$f_{McD}(x) = \frac{\alpha\eta\lambda\phi x^{-\phi-1}}{\mathrm{B}(a,b)}(1+\lambda x^{-\phi})^{-\eta a\alpha-1}\left[1-(1+\lambda x^{-\phi})^{-\alpha\eta}\right]^{b-1} \tag{43}$$

$$f_{EGD}(x) = \frac{\eta\lambda\phi\theta\alpha x^{-\phi-1}\left[1-(1+\lambda x^{-\phi})^{-\eta}\right]^{\alpha-1}\left\{1-\left[1-(1+\lambda x^{-\phi})^{-\eta}\right]^{\alpha}\right\}^{\theta-1}}{x^{\phi+1}} \tag{44}$$

$$f_{EKD}(x) = \alpha\eta\theta\lambda\phi x^{-\phi-1}(1+\lambda x^{-\phi})^{-\alpha-1}\left[1-(1+\lambda x^{-\phi})^{-\alpha}\right]^{\eta-1}\times\left\{1-\left[1-(1+\lambda x^{-\phi})^{-\alpha}\right]^{\eta}\right\}^{\theta-1} \tag{45}$$

The comparison is done using some measures of goodness of fit such as the Akaike information criterion (AIC), Bayesian information criterion (BIC), Consistent Akaike information criterion (CAIC), Kolmogrov-Smirnov (K-S) statistics and Cramér-von Misses (W*) Statistic. In general, the smaller the values of AIC, BIC, CAIC, K-S and W* the better the fit to the data.

Table 1. Parameter estimates and their standard errors (in parentheses) of the appliances data

Distribution	$\hat{\eta}$	$\hat{\lambda}$	$\hat{\phi}$	$\hat{\alpha}$	$\hat{\theta}$	$\hat{a}$	$\hat{b}$
Zubair-Dagum	0.686 (0.422)	83.346 (70.823)	0.847 (0.086)	3.998 (1.679)			
***Dagum**	1495.5 (1.058×10^{-7})	0.018 (0.0062)	0.509 (0.056)				
***Mc-Dagum**	1.275 (6.875)	1.427 (0.092)	3.455 (0.212)	500.56 (6.796)		10.505 (56.906)	0.064 (0.012)
***Exponentiated Generalized Dagum**	0.404 (0.044)	7.977 (0.651)	3.570 (0.391)	0.130 (0.021)	15.862 (5.196)		
***Exponentiated Kumaraswarmy-Dagum**	5.562 (1.517)	12.683 (2.158)	3.716 (0.755)	0.128 (0.029)	11.609 (3.922)		

**Nasiru et al. [12]*

Table 2. Log-likelihood, AIC, BIC, CAIC, K-S, and W* of the appliances data

Distribution	Log-likelihood	AIC	CAIC	BIC	K-S	W
Zubair-Dagum	**-330.227**	**668.453**	**669.743**	**674.787**	**0.209**	**0.577**
***Dagum**	-339.610	685.225	685.646	689.976	0.257	0.858
***Mc-Dagum**	-356.480	724.955	728.950	734.456	0.347	0.986
***Exponentiated generalized Dagum**	-340.910	691.818	692.721	699.736	0.264	0.882
***Exponentiated Kumaraswarmy-Dagum**	-341.650	693.295	694.198	701.213	0.269	0.925

The parameter estimates and their percentage errors in bracket for the first dataset are presented in Table 1. shows the properties of the two datasets. Table 2 shows the goodness of fit of the different models for the first dataset. From the results, the ZD distribution performed better than the competing distributions.

Table 3. Parameter Estimates and their standard errors (in parentheses) of the yarn data

Distribution	$\hat{\eta}$	$\hat{\lambda}$	$\hat{\phi}$	$\hat{\alpha}$	$\hat{\theta}$	$\hat{a}$	$\hat{b}$
Zubair-Dagum	2.513 (1.606)	75.112 (58.907)	1.475 (0.091)	4.101 (1.153)			
***Dagum**	11.599 (5.008)	19.749 (10.814)	1.126 (0.069)				
***Mc-Dagum**	98.780 (2.180×10^{-5})	0.027 (1.848×10^{-2})	0.600 (9.647×10^{-2})	46.276 (4.654×10^{-5})		0.333 (1.504×10^{-1})	25.042 (4.507×10^{-4})
***Exponentiated Generalized Dagum**	10.480 (13.022)	1.992 (0.251)	4.733 (0.587)	0.223 (0.032)	75.487 (27.669)		
***Exponentiated Kumaraswarmy-Dagum**	46.109 (1.295)	39.413 (5.006)	5.188 (0.961)	0.203 (0.040)	31.169 (11.023)		

Table 4. Log-likelihood, AIC, BIC, CAIC, K-S, and W*of the yarn data

Distribution	Log-likelihood	AIC	CAIC	BIC	K-S	W*
Zubair-Dagum	**-640.697**	**1289.393**	**1289.814**	**1299.814**	**0.136**	**0.607**
***Dagum**	-649.260	1304.517	1304.938	1312.333	0.164	0.821
***Mc-Dagum**	-628.200	1268.399	1269.616	1284.030	0.128	0.285
***Exponentiated generalized Dagum**	-653.070	1316.137	1317.040	1329.163	0.172	0.948
***Exponentiated Kumaraswarmy-Dagum**	-653.960	1318.816	1318.816	1330.938	0.178	0.985

Nasiru et al. (2017)

Table 3 shows the parameter estimates and their standard error in bracket for the second dataset, while Table 4 shows the goodness of fit of the different models for the second dataset. From the results, the ZD distribution outperformed the other competing distributions.

6 Conclusion

In this article, we have been able to develop a new Dagum distribution based on the Zubair-G family of distributions called the Zubair-Dagum (ZD) distribution. Furthermore, we derived and studied some major mathematical properties of the ZD distributions such as the quantile function, Moments and Moment generating function, Order statistics and Entropy. The parameters of the ZD distribution was also obtained and estimated using the method of maximum likelihood. From the results, the plots of the p.d.f. of the ZD distribution is right skewed while the plots of the hazard rate function of the ZD distribution displayed a variety of bathtub shapes and monotonically decreasing shapes while the survival function showed a monotonically decreasing function. The flexibility and superiority of the ZD distribution was examined using two real life datasets among some competing distributions and it showed superiority and more flexibility.

Competing Interests

Authors have declared that no competing interests exist.

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