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On Second Order Slope Rotatable Designs under Intra-class Correlated Structure of Errors Using a Pair of Dissimilar Incomplete Block Designs

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Authors' contributions

This work was carried out in collaboration between both authors. Author SB designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Author RVBB managed the analyses of the study and the literature searches. Both authors read and approved the final manuscript.

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Abstract

In this paper, a study of second order slope rotatable designs under intra-class correlation error structure using two suitably chosen dissimilar incomplete block designs like balanced incomplete block designs and symmetrical unequal block arrangements with two unequal block sizes are suggested. Further, we study the variance of the estimated slopes for different values of the intra-class correlation coefficient (ρ) and the distance from the centre (d) for v factors are suggested. Some illustrative examples are also suggested.

Keywords: Second order response surface designs; intra-class correlated errors.

1 Introduction

In the context of response surface methodology, Box and Hunter [1] introduced the concept of rotatability assuming errors are uncorrelated and homoscedastic. Das and Narasimham [2] constructed rotatable designs

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through balanced incomplete block designs (BIBD). Narasimham et al. [3] constructed rotatable designs through a pair of incomplete block designs (IBD). Hader and Park [4] constructed slope rotatable central composite designs. Victorbabu and Narasimham [5,6] constructed second order slope rotatable designs (SOSRD) through BIBD and a pair of IBD respectively. Victorbabu [7,8] constructed SOSRD using symmetrical unequal block arrangements (SUBA) with two unequal block sizes and suggested a review on SOSRD respectively. Victorbabu et al. [9] studied SOSRD using a pair of dissimilar IBD.

So far all the authors studied rotatable and slope rotatable designs assuming errors to be uncorrelated and homoscedastic. However, it is not uncommon to come across the practical situations when errors are correlated, violating usual assumptions. Das [10-12] introduced and studied robust second order rotatable designs (RSORD). Das [13] studied slope rotatability with correlated errors. Rajyalashmi [14] constructed second order rotatable and slope rotatable designs under different correlated error structures, Rajyalakshmi and Victorbabu [15-17] developed SOSRD under intra-class correlation structure of errors using central composite designs, symmetrical unequal block arrangements (SUBA) with two unequal block sizes and BIBD respectively. Sulochana and Victorbabu [18-20] studied SOSRD under intra-class correlated structure of errors using a pair of BIBD, SUBA with two unequal block sizes and partially balanced incomplete block type designs.

In this paper following the works of Das [13], Rajyalakshmi and Victorbabu [15-17] a study on SOSRD under intra-class correlation error structure using a pair of dissimilar incomplete block designs are suggested. Further, we study the variance function of the estimated slopes for different values of intra-class correlated coefficient (ρ) and also obtain the distance from centre (d) for 'v' (v number of factors).

2 Conditions for SOSRD under Intra-class Correlated Structure of Errors (cf. Das [13], Rajyalakshmi and Victorbabu [15-17]

A second order response surface design $D=\left(\left(X_{iu}\right)\right)$ for fitting,

$$y_u\left(x\right)=b_0+\sum_{i=1}^{v} b_i x_{iu}+\sum_{i=1}^{v} b_{ii} x_{iu}^2+\sum_{i=1}^{v}\sum_{i<j}^{v} b_{ij} x_{iu} x_{ju}+\varepsilon_u \quad (2.1)$$

where x_{iu} denotes the level of the i^{th} factor (i=1,2,…,v) in the u^{th} run (u=1,2,…,N) of the experiment, e_u's are correlated random errors, is said to be a SOSRD under intra-class correlated structure of errors, if the variance of the estimate of first order partial derivative of $y_u\left(x_{1u},x_{2u},x_{3u},...,x_{vu}\right)$ with respect to each independent variable X_i is only a function of the distance $\left(d^2=\sum_{i=1}^{v} X_i^2\right)$ of the point $\left(x_{1u},x_{2u},x_{3u},...,x_{vu}\right)$ from the origin (centre of the design). Such a spherical variance function for estimation of slopes in the second order response surface is achieved if the design points satisfy the following conditions (cf. (Das [13], Rajyalakshmi and Victorbabu [15-17].

The necessary and sufficient conditions for SOSRD under intra-class correlation error structure are

$$\sum_{u=1}^{N} x_{iu}=0,\ \sum_{u=1}^{N} x_{iu}x_{ju}=0,\ \sum_{u=1}^{N} x_{iu}x_{ju}^2=0,\ \sum_{u=1}^{N} x_{iu}x_{ju}x_{ku}=0,\ \sum_{u=1}^{N} x_{iu}^3=0,$$
$$\sum_{u=1}^{N} x_{iu}x_{ju}^3=0,\ \sum_{u=1}^{N} x_{iu}x_{ju}x_{ku}^2=0,\ \sum_{u=1}^{N} x_{iu}x_{ju}x_{ku}x_{lu}=0,\ \text{for } i\neq j\neq k\neq l; \quad (2.2)$$

$$\sum_{u=1}^{N} x_{iu}^2 = \text{constant} = N\theta_2 \tag{2.3}$$

$$\sum_{u=1}^{N} x_{iu}^4 = \text{constant} = cN\theta_4, \text{ for all } i \tag{2.4}$$

$$\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = \text{constant} = N\theta_4, \text{ for all values } i \neq j \tag{2.5}$$

From (2.4) and (2.5), we have,

$$\sum_{u=1}^{N} x_{iu}^4 = c \sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 \tag{2.6}$$

where c, θ_2 and θ_4 are constants. The summation is over the designs points.

Using the above simple symmetric conditions, the variances and covariances of the estimated parameters under the intra-class correlation error structure are as follows:

$$V\left(\hat{b}_0\right) = \frac{[\theta_4\left(c+v-1\right)A - v\rho N\theta_2^2]\sigma^2 A}{N\Delta} \tag{2.7}$$

$$V\left(\hat{b}_i\right) = \frac{\sigma^2(1-\rho)}{N\theta_2} \tag{2.8}$$

$$V\left(\hat{b}_{ij}\right) = \frac{\sigma^2(1-\rho)}{N\theta_4} \tag{2.9}$$

$$V\left(\hat{b}_{ii}\right) = \frac{\sigma^2(1-\rho)\theta_4\left(c+v-2\right)A-(v-1)\rho N\theta_2^2-(v-1)\theta_2^2(1-\rho)]}{\left(c-1\right)N\theta_4\Delta} \tag{2.10}$$

$$Cov\left(\hat{b}_0,\hat{b}_{ii}\right) = \frac{-\sigma^2\theta_2^2(1-\rho)A}{N\Delta} \tag{2.11}$$

$$Cov\left(\hat{b}_{ii},\hat{b}_{ij}\right) = \frac{\sigma^2(1-\rho)[\theta_2^2\left(1-\rho\right)-\theta_4 A+\rho N\theta_2^2]}{\left(c-1\right)N\theta_4\Delta} \tag{2.12}$$

Where $A=\left\{1+\left(N-1\right)\rho\right\}, \Delta=\left[\theta_4\left(c+v-1\right)A - v\rho N\theta_2^2 - v\theta_2^2\left(1-\rho\right)\right]$,

and the other covariances are zero.

An inspection of the variance of $\hat{b}_0$ shows that a necessary condition for the existence of a non-singular SOSRD under intra-class correlation error structure is

$$\theta_4 (c+v-1) A - v\rho N\theta_2^2 - v\theta_2^2 (1-\rho) > 0 \tag{2.13}$$

From (2.13), we have,

$$\frac{\theta_4}{\theta_2^2} > \frac{v}{c+v-1} \text{ (non-singularity condition)} \tag{2.14}$$

If the non-singularity condition (2.14) exists then only the design exists.

For the second order model,

$$\frac{\partial \hat{y}_u}{\partial x_i} = \hat{b}_i + 2\hat{b}_{ii} x_i + \sum_{i=1, j\neq i}^{v} \hat{b}_{ij} x_j,$$

$$V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right) = V\left(\hat{b}_i\right) + 4x_i^2 V\left(\hat{b}_{ii}\right) + \sum_{i=1, j\neq i}^{v} x_j^2 V\left(\hat{b}_{ij}\right) \tag{2.15}$$

The condition for right hand side of equation (2.15) to be a function of $(d^2 = \sum_{i=1}^{v} X_i^2)$ alone (for slope rotatability) is clearly,

$$V\left(\hat{b}_{ii}\right) = \frac{1}{4} V\left(\hat{b}_{ij}\right) \tag{2.16}$$

(cf. Hader and Park [4])

On simplification of (2.16) using (2.9) and (2.10) leads to

$$\left(\frac{AcN\theta_4 - B}{1-\rho}\right)\left[4N - \left(\frac{AcN\theta_4 - B}{A\theta_4}\right) v \left(\frac{N\theta_2^2 (1-\rho)}{A\theta_4}\right) - (v-2)\left(\frac{AN\theta_4 - B}{A\theta_4}\right)\right] +$$
$$\left(\frac{AN\theta_4 - B}{1-\rho}\right)\left[4(v-2) - (v-1)\left(\frac{AN\theta_4 - B}{AN\theta_4}\right)\right] - N^2\theta_2^2\left[4(v-1) + v\left(\frac{AN\theta_4 - B}{AN\theta_4}\right)\right] = 0 \tag{2.17}$$

$$A = \{1 + (N-1)\rho\},\ B = \rho N^2 \theta_2^2$$

On simplification of (2.17) we get

$$\left(\{1+(N-1)\rho\}^2\right)\left[\theta_4\left[v(5-c)-(c-3)^2\right]+\theta_2^2\left[v(c-5)+4\right]\right]=0 \tag{2.18}$$

If ρ = 0 condition (2.18) equal to

$$\theta_4\left[v(5-c)-(c-3)^2\right]+\theta_2^2\left[v(c-5)+4\right]=0 \tag{2.19}$$

This is similar to the SOSRD condition of Victorbabu and Narasimham [6].

Therefore, equations (2.2) to (2.12), (2.14) to (2.18) give a set of conditions for SOSRD under intra-class correlation error structure for any general second order response surface design. Further,

$$V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right)=\frac{1}{N}\left(\frac{1}{\theta_2}+\frac{d^2}{\theta_4}\right)(1-\rho)\sigma^2 \tag{2.20}$$

3 Construction of SOSRD under Intra-class Correlated Structure of Errors Using a Pair of Dissimilar Incomplete Block Designs

Following the methods of construction of Das [13], Victorbabu and Narasimham [5,6], Victorbabu [8], Victorbabu et al. [9] Rajyalakshmi and Victorbabu [15-17], here a study on SOSRD with intra-class correlation error structure using two suitably chosen dissimilar incomplete block designs like BIBD and SUBA with two unequal block sizes are studied. Let ρ be the correlation errors of any two observations, each having the same σ^2. In this paper, we follow the notations of Das and Narasimham [2], Narasimham et al. [3], and Victorbabu and Narasimham [5,6].

Let $D_1=(v,b_1,r_1,k_1,\lambda_1)$ denote a BIBD with $r_1 \lesseqgtr c\lambda_1$ and $D_2=(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2)$ denote a SUBA with two unequal block sizes (where b_{21} blocks each of size k_{21} and b_{22} blocks each of size k_{22}) with $b_{21}+b_{22}=b_2$, $k_2=\max(k_{21},k_{22})$ and $r_2 \gtreqless c\lambda_2$ in v-treatments respectively. Let $2^{t(k_1)}$ and $2^{t(k_2)}$ denote fractional replicate of 2^{k_1} and 2^{k_2} with levels ± 1 levels in which no interaction with less than five factors is confounded. $\left[1-(v,b_1,r_1,k_1,\lambda_1)\right]2^{t(k_1)}$ denote the design points generated from the transpose of incidence matrix of the design D_1 by "multiplication" [21]. Let $\left[1-(v,b_1,r_1,k_1\lambda_1)\right]2^{t(k_1)}$ are the $b_1 2^{t(k_1)}$ design points generated from D_1 by "multiplication" $\left[\alpha-(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2)\right]2^{t(k_2)}$ are the $b_2 2^{t(k_2)}$ design points generated from D_2 by 'multiplication'. Let n_o be the number of central points in the design. The method of construction of SOSRD under intra-class correlation error structure is given in the following theorem.

3.1 Theorem

Let $D_1=\left(v,b_1,r_1,k_1,\lambda_1\right)$ and $D_2=\left(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2\right)$ are two dissimilar incomplete block designs, $2^{t\left(k_1\right)}$ and $2^{t\left(k_2\right)}$ factorials with levels ± 1 and n_o is pre-fixed points, then the design points,

$$\left[1-\left(v,b_1,r_1,k_1,\lambda_1\right)\right]2^{t\left(k_1\right)}\cup\left[\alpha-\left(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2\right)\right]2^{t\left(k_2\right)}\cup n_0$$

give a v-dimensional SOSRD under intra-class correlation error structure in $N=b_1 2^{t\left(k_1\right)}+b_2 2^{t\left(k_2\right)}+n_0$ design points, where α^2 is a positive real root of the biquadratic equation,

$$\begin{aligned}
&\left[2^{2t\left(k_2\right)}N\left\{r_2\lambda_2\left(6-v\right)-r_2^2+\lambda_2^2\left(5v-9\right)\right\}+2^{3t\left(k_2\right)}r_2^2\left(vr_2-5v\lambda_2+4\lambda_2\right)\right]A^2\alpha^8+\\
&\left[2^{t\left(k_1\right)+2t\left(k_2\right)+1}r_1r_2\left(vr_2-5v\lambda_2+4\lambda_2\right)\right]A^2\alpha^6+\\
&\left[\begin{aligned}&2^{t\left(k_1\right)+t\left(k_2\right)}N\left\{\lambda_1\lambda_2\left(10v-18\right)+\left(6-v\right)\left(r_1\lambda_2+r_2\lambda_1\right)-2r_1r_2\right\}+\\&2^{t\left(k_1\right)+2t\left(k_2\right)}r_2^2\left(vr_1-5v\lambda_1+4\lambda_1\right)+2^{2t\left(k_1\right)+t\left(k_2\right)}r_1^2\left(vr_2-5v\lambda_2+4\lambda_2\right)\end{aligned}\right]A^2\alpha^4+\\
&\left[2^{2t\left(k_1\right)+t\left(k_2\right)+1}r_1r_2\left(vr_1-5v\lambda_1+4\lambda_1\right)\right]A^2\alpha^2+\\
&\left[2^{2t\left(k_1\right)}N\left\{\lambda_1^2\left(5v-9\right)+r_1\lambda_1\left(6-v\right)-r_1^2\right\}+2^{3t\left(k_1\right)}r_1^2\left(vr_1-5v\lambda_1+4\lambda_1\right)\right]A^2=0
\end{aligned}\tag{3.1}$$

where $A=\{1+(N-1)\rho\}$

Proof: For the design points generated from a pair of dissimilar IBD, simply symmetry conditions (2.2) are true. Further, from (2.3), (2.4) and (2.5) we have,

$$\sum_{u=1}^{N}x_{iu}^2=r_1 2^{t(k_1)}+r_2 2^{t\left(k_2\right)}\alpha^2=N\theta_2,\text{forall } i \tag{3.2}$$

$$\sum_{u=1}^{N}x_{iu}^4=r_1 2^{t(k_1)}+r_2 2^{t\left(k_2\right)}\alpha^4=cN\theta_4,\text{forall } i \tag{3.3}$$

$$\sum_{u=1}^{N}x_{iu}^2x_{ju}^2=\lambda_1 2^{t(k_1)}+\lambda_2 2^{t\left(k_2\right)}\alpha^4=N\theta_4,\text{forall } i\neq j \tag{3.4}$$

Substituting θ_2, θ_4 and c in condition (2.18) and on simplification, we get (3.1). The design exists only if at least one positive real root exits for equation (3.1). Solving equation (3.1) we get the SOSRD with intra-class correlation error structure using a pair of dissimilar IBD with different 'α' values for the 'v' different factors.

The variance of estimated slopes of these SOSRD for $0 \le \rho \le 0.9$ and for $12 \le v \le 16$ factors are given in Table 1.

Example: We illustrate the above the method with construction of SOSRD with intra-class correlation error structure for v=12-factors with the help of a pair of dissimilar incomplete block designs with parameters $D_1=\left(v=12, b_1=44, r_1=11, k_1=3, \lambda_1=2\right)$,
$D_2=\left(v=12, b_2=26, r_2=6, k_{21}=2, k_{22}=3, b_{21}=6, b_{22}=20, \lambda_2=1\right)$ is given below.

The design points,

$$\left[1\text{-}\left(v=12, b_1=44, r_1=11, k_1=3, \lambda_1=2\right)\right]2^3U$$
$$\left[\alpha\text{-}\left(v=12, b_2=26, r_2=6, k_{21}=2, k_{22}=3, b_{21}=6, b_{22}=20, \lambda_2=1\right)\right]2^3Un_0$$

will give SOSRD under intra-class correlation error structure in N = 561 design points for 12 factors. We have,

$$\sum_{u=1}^{N} x_{iu}^2 = 88+48\alpha^2 = N\theta_2 \tag{3.5}$$

$$\sum_{u=1}^{N} x_{iu}^4 = 88+48\alpha^4 = cN\theta_4 \tag{3.6}$$

$$\sum_{u=1}^{N} x_{iu}^2 x_{ju}^2 = 16+8\alpha^4 = N\theta_4 \tag{3.7}$$

From (3.6) and (3.7), we get $c=\frac{88+48\alpha^4}{16+8\alpha^4}$. Substituting for θ_2, θ_4 and c in (2.18) and on simplification, we get the following biquadratic equation in α^2.

$$(1+560\rho)^2\left(459072\alpha^8-1081344\alpha^6+1009792\alpha^4-1351680\alpha^2+520256\right)=0 \tag{3.8}$$

Equation (3.8) has only one positive real root $\alpha^2=1.8478$ ($(\forall(0<\rho<1))$).This can be alternatively written directly from equation (3.1). Solving (3.8), we get $\alpha=1.3593\ \forall\rho$. Substituting this value of ‘α’ in (3.5), (3.6) and (3.7) we obtain $\theta_2=0.3149$, $\theta_4=0.0772$, $c=5.5183$. It can be verified that non-singularity condition (2.14) is also satisfied. From (2.7) to (2.12), we can obtain the variances and covariances. Further from (2.20), we have,

$$V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right)=\left(0.0057+0.0230d^2\right)(1-\rho)\sigma^2 \tag{3.9}$$

4 A Study on Dependence of the Variance Function of the Response at Different Design Points

Here, we study the dependence of variance function of response at different designs points of SOSRD under intra-class correlation error structure using a pair of dissimilar incomplete block designs. Given 'v' factors different values of intra-class correlation coefficient 'ρ , and distance from centre'd' between 0 and 1, the variances are tabulated. From (3.9) the variance of the estimated derivative is obtained by

$$V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right)=0.0053(\text{taking } \rho=0.1,\ d=0.1,\ \sigma=1)$$

The numerical calculations are provided in Table 1 and Table 2.

Table 1. The variances of estimated derivatives (slopes) for the $12 \le v \le 16$ factors

ρ	**(v=12, b_1 =44, r_1=11, k_1= 3, λ_1=2) (v=12, b_2=26, r_1= 6, k_{11}=2, k_{12}=3, b_{11}=6, b_{12}=20, λ_2=1) α= 1.8822, N =561**	**(v=12, b_1 =44, r_1=11, k_1= 3, λ_1=2) (v=12, b_2=26, r_1= 6, k_{11}=2, k_{12}=3, b_{11}=6, b_{12}=20, λ_2=1), α= 1.3593, N =737**
	$V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right)$	$V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right)$
0	$0.0057\sigma^2 +0.0231d^2\sigma^2$	$0.0029\ \sigma^2 +0.0067\ d^2\sigma^2$
0.1	$0.0051\ \sigma^2 +0.0208\ d^2\sigma^2$	$0.0026\ \sigma^2 +0.0061\ d^2\sigma^2$
0.2	$0.0045\ \sigma^2 +0.0185\ d^2\sigma^2$	$0.0023\ \sigma^2 +0.0054\ d^2\sigma^2$
0.3	$0.0039\ \sigma^2 +0.0161\ d^2\sigma^2$	$0.0020\ \sigma^2 +0.0047\ d^2\sigma^2$
0.4	$0.0034\ \sigma^2+0.0139\ d^2\sigma^2$	$0.0017\ \sigma^2 +0.0040\ d^2\sigma^2$
0.5	$0.0028\ \sigma^2+0.0115\ d^2\sigma^2$	$0.0014\ \sigma^2 +0.0033\ d^2\sigma^2$
0.6	$0.0023\ \sigma^2 +0.0062\ d^2\sigma^2$	$0.0012\ \sigma^2 +0.0027\ d^2\sigma^2$
0.7	$0.0017\ \sigma^2 +0.0069\ d^2\sigma^2$	$0.0008\ \sigma^2 +0.0020\ d^2\sigma^2$
0.8	$0.0011\ \sigma^2+0.0046\ d^2\sigma^2$	$0.0005\ \sigma^2 +0.0013\ d^2\sigma^2$
0.9	$0.0005\ \sigma^2+0.0023\ d^2\sigma^2$	$0.0003\ \sigma^2 +0.0007\ d^2\sigma^2$

ρ	**(v=12, b_1 =44, r_1=11, k_1= 6, λ_1=5) (v=12, b_2=26, r_1= 6, k_{11}=2, k_{12}=3, b_{11}=6, b_{12}=20, λ_2=1), α= 2.6815, N =913**	**(v=16, b_1 =48, r_1=15, k_1= 5, λ_1=4) (v=16, b_2=28, r_1= 6, k_2=4, b_{11}=12, b_{12}=16, λ_2=1), α= 2.6124, N =1217**
	$V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right)$	$V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right)$
0	$0.0014\ \sigma^2+0.0017\ d^2\sigma^2$	$0.0011\ \sigma^2+0.0012\ d^2\sigma^2$
0.1	$0.0013\ \sigma^2+0.0016\ d^2\sigma^2$	$0.0010\ \sigma^2+0.0011\ d^2\sigma^2$
0.2	$0.0011\ \sigma^2+0.0013\ d^2\sigma^2$	$0.0009\ \sigma^2+0.0009\ d^2\sigma^2$
0.3	$0.0010\ \sigma^2+0.0012\ d^2\sigma^2$	$0.0008\ \sigma^2+0.0009\ d^2\sigma^2$
0.4	$0.0009\ \sigma^2+0.0010\ d^2\sigma^2$	$0.0007\ \sigma^2+0.0008\ d^2\sigma^2$
0.5	$0.0007\ \sigma^2+0.0009\ d^2\sigma^2$	$0.0006\ \sigma^2+0.0006\ d^2\sigma^2$
0.6	$0.0006\ \sigma^2+0.0007\ d^2\sigma^2$	$0.0004\ \sigma^2+0.0005\ d^2\sigma^2$
0.7	$0.0004\ \sigma^2+0.0005\ d^2\sigma^2$	$0.0004\ \sigma^2+0.0004\ d^2\sigma^2$
0.8	$0.0003\ \sigma^2+0.0003\ d^2\sigma^2$	$0.0002\ \sigma^2+0.0002\ d^2\sigma^2$
0.9	$0.0001\ \sigma^2+0.0002\ d^2\sigma^2$	$0.0001\ \sigma^2+0.0001\ d^2\sigma^2$

ρ	**(v=16, b_1 =, r_1=6, k_1= 6, λ_1=2) (v=16, b_2=28, r_1= 6, k_2=4, b_{11}=12, b_{12}=16, λ_2=1), α= 1.8394, N =961**	**(v=16, b_1 =48, r_1=1, k_1= 5, λ_1=4) (v=16, b_2=36, r_1= 7, k_{11}=4, k_{12}=3, b_{11}=4, b_{12}=32, λ_2=1), α= 1.5929, N =1345**
	$V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right)$	$V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right)$
0	$0.0019\,\sigma^2+0.0040\,d^2\sigma^2$	$0.0013\sigma^2+0.0043\,d^2\sigma^2$
0.1	$0.0017\,\sigma^2+0.0036d^2\sigma^2$	$0.0011\sigma^2+0.0039\,d^2\sigma^2$
0.2	$0.0015\,\sigma^2+0.0032\,d^2\sigma^2$	$0.0010\,\sigma^2+0.0035\,d^2\sigma^2$
0.3	$0.0014\,\sigma^2+0.0028\,d^2\sigma^2$	$0.0009\,\sigma^2+0.0030\,d^2\sigma^2$
0.4	$0.0011\,\sigma^2+0.0024\,d^2\sigma^2$	$0.0008\,\sigma^2+0.0026\,d^2\sigma^2$
0.5	$0.0009\,\sigma^2+0.0020\,d^2\sigma^2$	$0.0007\,\sigma^2+0.0022\,d^2\sigma^2$
0.6	$0.0008\,\sigma^2+0.0016\,d^2\sigma^2$	$0.0005\sigma^2+0.0017\,d^2\sigma^2$
0.7	$0.0006\,\sigma^2+0.0012\,d^2\sigma^2$	$0.000\,4\sigma^2+0.0013d^2\sigma^2$
0.8	$0.0004\,\sigma^2+0.0008\,d^2\sigma^2$	$0.0003\,\sigma^2+0.0009\,d^2\sigma^2$
0.9	$0.0001\,\sigma^2+0.0004\,d^2\sigma^2$	$0.0001\,\sigma^2+0.0004\,d^2\sigma^2$

Table 2. Study of dependence of estimated slope of second order response surface design under intra-class correlation error structure using a pair of dissimilar incomplete block designs at different design points for $12 \le v \le 16$ factors for different values of ' ρ ', d and σ=1

((12, 44, 11, 3, 2), (12, 26, 6, 2, 3, 6, 20, 1)), a=1.3593, N=561										
ρ	**d=0.1**	**d=0.2**	**d=0.3**	**d=0.4**	**d=0.5**	**d=0.6**	**d=0.7**	**d=0.8**	**d=0.9**	**d=1**
0	0.0059	0.0066	0.0077	0.0094	0.0114	0.0139	0.0169	0.0204	0.0244	0.0287
0.1	0.0053	0.0059	0.0069	0.0085	0.0102	0.0126	0.0153	0.0183	0.0219	0.0259
0.2	0.0047	0.0053	0.0062	0.0075	0.0091	0.0112	0.0136	0.0163	0.0195	0.0229
0.3	0.0041	0.0046	0.0054	0.0065	0.0080	0.0098	0.0119	0.0143	0.0171	0.0201
0.4	0.0035	0.0039	0.0046	0.0056	0.0069	0.0084	0.0102	0.0122	0.0146	0.0172
0.5	0.0029	0.0033	0.0039	0.0047	0.0057	0.0069	0.0085	0.0102	0.0122	0.0144
0.6	0.0024	0.0026	0.0031	0.0037	0.0046	0.0056	0.0068	0.0082	0.0097	0.0115
0.7	0.0018	0.0019	0.0023	0.0028	0.0034	0.0042	0.0051	0.0061	0.0073	0.0086
0.8	0.0012	0.0013	0.0015	0.0018	0.0023	0.0028	0.0034	0.0041	0.0049	0.0057
0.9	0.0006	0.0007	0.0008	0.0009	0.0011	0.0014	0.0017	0.0020	0.0024	0.0029

((12, 33, 11, 4, 3), (12, 26, 6, 2, 3, 6, 20, 1)), a=1.8822, N=737										
ρ	**d=0.1**	**d=0.2**	**d=0.3**	**d=0.4**	**d=0.5**	**d=0.6**	**d=0.7**	**d=0.8**	**d=0.9**	**d=1**
0	0.0029	0.0032	0.0035	0.0039	0.0045	0.0053	0.0062	0.0072	0.0083	0.0096
0.1	0.0026	0.0028	0.0031	0.0035	0.0041	0.0048	0.0056	0.0065	0.0075	0.0087
0.2	0.0023	0.0025	0.0028	0.0031	0.0037	0.0043	0.0049	0.0058	0.0068	0.0077
0.3	0.0020	0.0022	0.0024	0.0028	0.0032	0.0037	0.0043	0.0050	0.0058	0.0067
0.4	0.0018	0.0019	0.0021	0.0024	0.0027	0.0032	0.0037	0.0043	0.0050	0.0058
0.5	0.0015	0.0016	0.0017	0.0019	0.0023	0.0027	0.0031	0.0036	0.0042	0.0048
0.6	0.0012	0.0013	0.0014	0.0016	0.0018	0.0021	0.0025	0.0029	0.0033	0.0039
0.7	0.0009	0.0009	0.0010	0.0012	0.0014	0.0016	0.0019	0.0022	0.0025	0.0029
0.8	0.0006	0.0006	0.0007	0.0008	0.0009	0.0010	0.0012	0.0014	0.0017	0.0019
0.9	0.0003	0.0003	0.0003	0.0004	0.0005	0.0005	0.0006	0.0007	0.0008	0.0009

((12, 22, 11, 6, 5), (12, 26, 6, 2, 3, 6, 20, 1)), a=2.6815, N=913										
ρ	**d=0.1**	**d=0.2**	**d=0.3**	**d=0.4**	**d=0.5**	**d=0.6**	**d=0.7**	**d=0.8**	**d=0.9**	**d=1**
0	0.0015	0.0015	0.0016	0.0017	0.0018	0.0021	0.0023	0.0025	0.0028	0.0032
0.1	0.0013	0.0014	0.0014	0.0015	0.0017	0.0019	0.0021	0.0023	0.0025	0.0029
0.2	0.0011	0.0012	0.0013	0.0014	0.0015	0.0016	0.0018	0.0020	0.0023	0.0025
0.3	0.0010	0.0011	0.0011	0.0012	0.0013	0.0014	0.0016	0.0017	0.0019	0.0022
0.4	0.0008	0.0009	0.0009	0.0010	0.0011	0.0012	0.0014	0.0015	0.0017	0.0019
0.5	0.0007	0.0008	0.0008	0.0009	0.0009	0.0010	0.0011	0.0013	0.0014	0.0016
0.6	0.0006	0.0006	0.0006	0.0007	0.0008	0.0008	0.0009	0.0010	0.0011	0.0013
0.7	0.0004	0.0005	0.0005	0.0005	0.0006	0.0006	0.0007	0.0007	0.0009	0.0009
0.8	0.0003	0.0003	0.0003	0.0003	0.0004	0.0004	0.0005	0.0005	0.0007	0.0006
0.9	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003

((16, 16, 6, 6, 2), (16, 28, 6, 4, 12, 16, 1)), a=1.8394, N=961										
ρ	**d=0.1**	**d=0.2**	**d=0.3**	**d=0.4**	**d=0.5**	**d=0.6**	**d=0.7**	**d=0.8**	**d=0.9**	**d=1**
0	0.0019	0.0021	0.0023	0.0026	0.0029	0.0034	0.0039	0.0045	0.0052	0.0059
0.1	0.0018	0.0019	0.0021	0.0023	0.0027	0.0031	0.0035	0.0041	0.0047	0.0054
0.2	0.0016	0.0017	0.0018	0.0021	0.0024	0.0027	0.0031	0.0036	0.0042	0.0048
0.3	0.0014	0.0015	0.0016	0.0018	0.0021	0.0024	0.0027	0.0032	0.0036	0.0042
0.4	0.0012	0.0013	0.0014	0.0015	0.0018	0.0020	0.0024	0.0027	0.0031	0.0036
0.5	0.0009	0.0010	0.0011	0.0013	0.0015	0.0017	0.0019	0.0023	0.0026	0.0029
0.6	0.0008	0.0008	0.0009	0.0010	0.0012	0.0014	0.0016	0.0018	0.0021	0.0024
0.7	0.0006	0.0006	0.0007	0.0008	0.0009	0.0010	0.0012	0.0014	0.0016	0.0018
0.8	0.0004	0.0004	0.0005	0.0005	0.0006	0.0007	0.0008	0.0009	0.0010	0.0012
0.9	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003	0.0004	0.0005	0.0005	0.0006

((16, 48, 15, 5, 4), (16, 28, 6, 4, 12, 16, 1)), a=2.6124, N=1217										
ρ	**d=0.1**	**d=0.2**	**d=0.3**	**d=0.4**	**d=0.5**	**d=0.6**	**d=0.7**	**d=0.8**	**d=0.9**	**d=1**
0	0.0011	0.0012	0.0012	0.0013	0.0014	0.0016	0.0017	0.0019	0.0021	0.0024
0.1	0.0010	0.0010	0.0011	0.0012	0.0013	0.0014	0.0016	0.0017	0.0019	0.0021
0.2	0.0009	0.0009	0.0009	0.0010	0.0011	0.0012	0.0014	0.0015	0.0017	0.0018
0.3	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	0.0012	0.0013	0.0015	0.0016
0.4	0.0007	0.0007	0.0007	0.0008	0.0008	0.0009	0.0010	0.0011	0.0013	0.0014
0.5	0.0006	0.0006	0.0006	0.0007	0.0007	0.0008	0.0009	0.0009	0.0011	0.0011
0.6	0.0005	0.0005	0.0005	0.0005	0.0006	0.0006	0.0007	0.0008	0.0008	0.0009
0.7	0.0003	0.0003	0.0004	0.0004	0.0004	0.0005	0.0005	0.0006	0.0006	0.0007
0.8	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003	0.0003	0.0004	0.0004	0.0005
0.9	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0001	0.0002	0.0002	0.0002

((16, 48, 15,5,4), (16, 36,7,4,3,4,32,1)), a=1.5929, N=1345										
ρ	**d=0.1**	**d=0.2**	**d=0.3**	**d=0.4**	**d=0.5**	**d=0.6**	**d=0.7**	**d=0.8**	**d=0.9**	**d=1**
0	0.0013	0.0015	0.0017	0.0020	0.0024	0.0029	0.0034	0.0041	0.0048	0.0056
0.1	0.0012	0.0013	0.0015	0.0018	0.0022	0.0026	0.0031	0.0037	0.0043	0.0051
0.2	0.0011	0.0011	0.0014	0.0016	0.0019	0.0022	0.0027	0.0033	0.0039	0.0045
0.3	0.0009	0.0010	0.0012	0.0014	0.0017	0.0020	0.0024	0.0028	0.0034	0.0039
0.4	0.0008	0.0009	0.0010	0.0012	0.0014	0.0017	0.0020	0.0024	0.0029	0.0034
0.5	0.0007	0.0007	0.0008	0.0010	0.0012	0.0014	0.0017	0.0020	0.0024	0.0028
0.6	0.0005	0.0006	0.0008	0.0008	0.0009	0.0011	0.0014	0.0016	0.0019	0.0023
0.7	0.0004	0.0004	0.0005	0.0006	0.0007	0.0008	0.0010	0.0012	0.0014	0.0017
0.8	0.0003	0.0003	0.0003	0.0004	0.0005	0.0005	0.0007	0.0008	0.0009	0.0011
0.9	0.0001	0.0001	0.0002	0.0002	0.0002	0.0003	0.0004	0.0004	0.0004	0.0006

5 Conclusions

From Table 1 and Table 2, we conclude that,

(i) For given v and ρ, $V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right)$ increases as d increases.

(ii) For given v and d, $V\left(\frac{\partial \hat{y}_u}{\partial x_i}\right)$ decreases as ρ increases

Competing Interests

Authors have declared that no competing interests exist.

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